UNIT 7

Similarity and Transformations

Here are some flags of different countries and Canadian provinces.
Some of these flags have line symmetry.
Picture each flag lying on your desk.
Which flags have a line of symmetry that is:

- vertical?
- horizontal?
- oblique?

Identify any flag that has more than one line of symmetry.
Which flag has the most lines of symmetry?

What You'll Learn

- Draw and interpret scale diagrams.
- Apply properties of similar polygons.
- Identify and describe line symmetry and rotational symmetry.

Why It's Important

Architects, engineers, designers, and surveyors use similarity and scale diagrams routinely in their work.
Symmetry can be seen in art and nature.
An understanding of symmetry helps us to appreciate and find out more about our world, and to create works of art.

Key Words

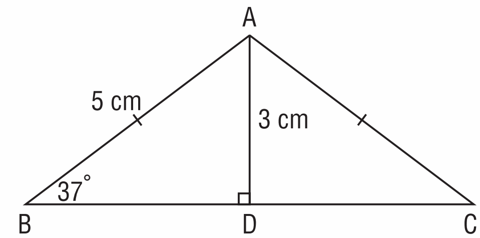
- scale diagram
- corresponding lengths
- scale factor
- proportion
- similar polygons
- corresponding angles
- corresponding sides
- rotational symmetry
- order of rotation
- angle of rotation symmetry

Start Where You Are

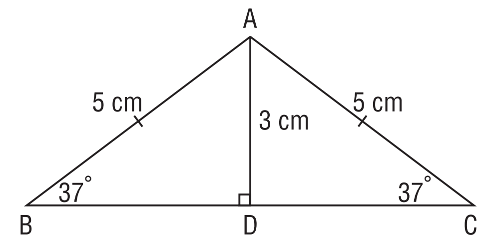
What Should I Recall?

Suppose I have to solve this problem:
Determine the unknown measures of the angles and sides in ΔABC.
The given measures are rounded to the nearest whole number.

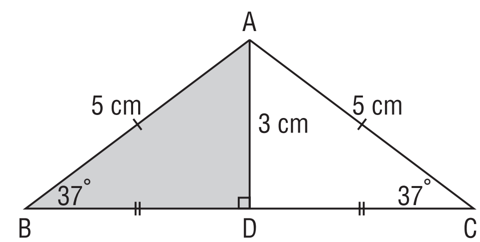
I think of what I already know about triangles.
I see that AB and AC have the same hatch marks; this means the sides are equal.
AC = AB
So, AC = 5 cm



I know that a triangle with 2 equal sides is an isosceles triangle.
So, ΔABC is isosceles.
An isosceles triangle has 2 equal angles that are formed where the equal sides intersect the third side.
I use 3 letters to describe an angle.
So, $\angle ACD = \angle ABD$
$= 37°$



Since ΔABC is isosceles, the height AD is the perpendicular bisector of the base BC.
So, BD = DC and $\angle ADB = 90°$
I can use the Pythagorean Theorem in ΔABD to calculate the length of BD.

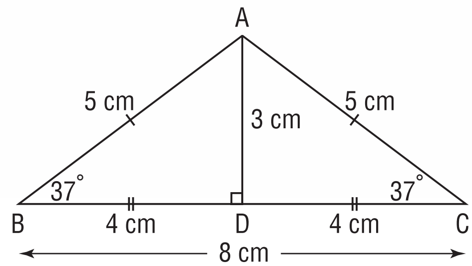


$$\begin{aligned} AD^2 + BD^2 &= AB^2 \\ 3^2 + BD^2 &= 5^2 \\ 9 + BD^2 &= 25 \\ 9 - 9 + BD^2 &= 25 - 9 \\ BD^2 &= 16 \\ BD &= \sqrt{16} \\ BD &= 4 \end{aligned}$$

BD = 4 cm
So, BC = 2 × 4 cm
= 8 cm

I know that the sum of the angles in a triangle is 180°.
So, I can calculate the measure of ∠BAC.

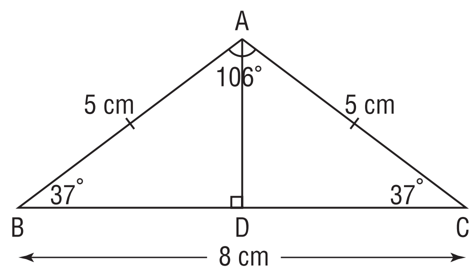
$\angle BAC + \angle ACD + \angle ABD = 180°$
$\angle BAC + 37° + 37° = 180°$
$\angle BAC + 74° = 180°$
$\angle BAC + 74° - 74° = 180° - 74°$
$\angle BAC = 106°$



My friend Janelle showed me a different way to calculate.
She recalled that the line AD is a line of symmetry for an isosceles triangle.
So, ΔABD is congruent to ΔACD.
This means that ∠BAD = ∠CAD
Janelle calculated the measure of ∠BAD in ΔABD.

$\angle BAD + 37° + 90° = 180°$
$\angle BAD + 127° = 180°$
$\angle BAD + 127° - 127° = 180° - 127°$
$\angle BAD = 53°$

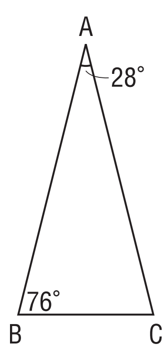
Then, $\angle BAC = 2 \times 53°$
$= 106°$



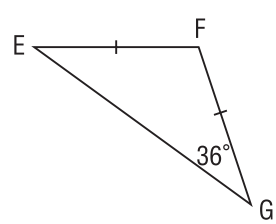
Check

1. Calculate the measure of each angle.

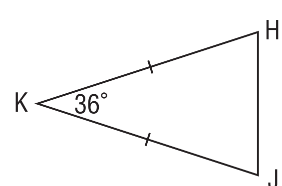
a) ∠ACB



b) ∠GEF and ∠GFE



c) ∠HJK and ∠KHJ

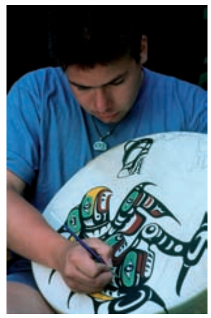


7.1 Scale Diagrams and Enlargements

FOCUS

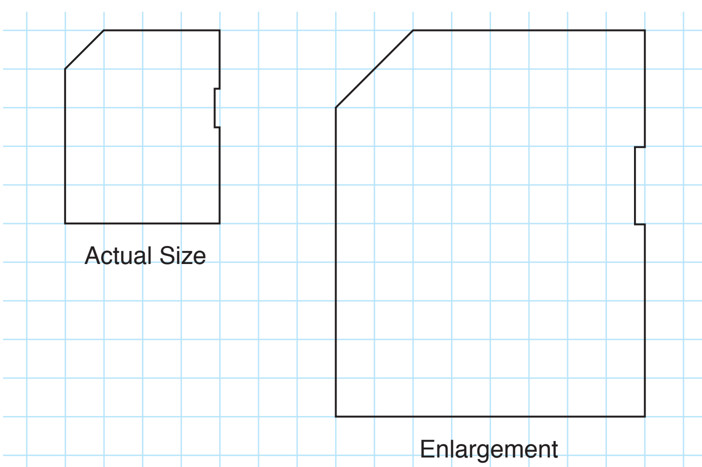
- Draw and interpret scale diagrams that represent enlargements.

How are these photos alike?
How are they different?



Investigate

You will need 0.5-cm grid paper.
Here is an actual size drawing of a memory card for a digital camera and an enlargement of the drawing.



➤ Copy the drawings on grid paper.
Measure the lengths of pairs of matching sides on the drawings.
Label each drawing with these measurements.

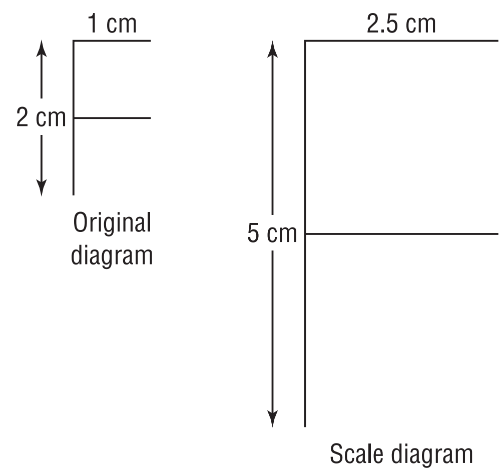
➤ For each measurement, write the fraction: $\frac{\text{Length on enlargement}}{\text{Length on actual size drawing}}$
Write each fraction as a decimal.
What do you notice about these numbers?

Reflect & Share

Compare your numbers with those of another pair of students.
Work together to draw a different enlargement of the memory card.
Determine the fraction $\frac{\text{Length on enlargement}}{\text{Length on actual size drawing}}$ for this new enlargement.

Connect

A diagram that is an enlargement or a reduction of another diagram is called a **scale diagram.**
Here is letter F and a scale diagram of it.



Compare the matching lengths in the scale diagram and in the original diagram.

$$\frac{\text{Length of vertical segment on the scale diagram}}{\text{Length of vertical segment on the original diagram}} = \frac{5\text{ cm}}{2\text{ cm}}$$
$$= 2.5$$
$$\frac{\text{Length of horizontal segment on scale diagram}}{\text{Length of horizontal segment on original diagram}} = \frac{2.5\text{ cm}}{1\text{ cm}}$$
$$= 2.5$$

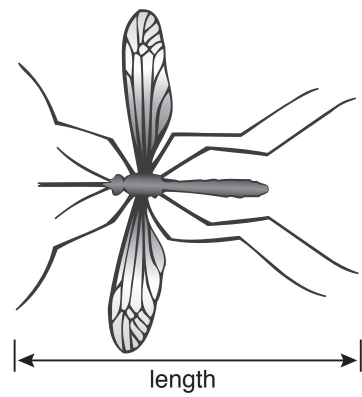
This equation is called a **proportion** because it is a statement that two ratios are equal.

Each length on the original diagram is multiplied by 2.5 to get the matching length on the scale diagram. Matching lengths on the original diagram and the scale diagram are called **corresponding lengths.**

The fraction $\frac{\text{Length on scale diagram}}{\text{Length on original diagram}}$ is called the **scale factor** of the scale diagram.
A scale factor can be expressed as a fraction or as a decimal.
For the diagram above, the scale factor is $\frac{5}{2}$, or 2.5.
Pairs of corresponding lengths have the same scale factor, so we say that corresponding lengths are *proportional.*
Each segment of the enlargement is longer than the corresponding segment on the original diagram, so the scale factor is greater than 1.

Example 1 Using Corresponding Lengths to Determine the Scale Factor

This drawing of a mosquito was printed in a newspaper article about the West Nile Virus.
The actual length of the mosquito is 12 mm.
Determine the scale factor of the diagram.



A Solution

Measure the length on the scale drawing of the mosquito, to the nearest millimetre.
The length is 4.5 cm, which is 45 mm.

To calculate the scale factor, the units of length must be the same.

The scale factor is: $\dfrac{\text{Length on scale diagram}}{\text{Length of mosquito}} = \dfrac{45\text{ mm}}{12\text{ mm}}$

$= 3.75$

The scale factor is 3.75.

Example 2 Using a Scale Factor to Determine Dimensions

This photo of longhouses has dimensions 9 cm by 6 cm.
The photo is to be enlarged by a scale factor of $\frac{7}{2}$.
Calculate the dimensions of the enlargement.

Solutions

To determine a length on the scale diagram, multiply the corresponding length on the original diagram by the scale factor.

Method 1

Use mental math.

Length of enlargement: $\frac{7}{2} \times 9 \text{ cm} = \frac{7 \times 9 \text{ cm}}{2}$

$= 31.5 \text{ cm}$

Width of enlargement: $\frac{7}{2} \times 6 \text{ cm} = \frac{7 \times 6 \text{ cm}}{2}$

$= 21 \text{ cm}$

The dimensions of the enlargement are 31.5 cm by 21 cm.

Method 2

Use a calculator.

Write $\frac{7}{2}$ as 3.5.

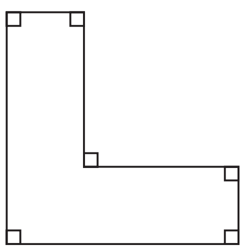
Length of enlargement: $3.5 \times 9 \text{ cm} = 31.5 \text{ cm}$

Width of enlargement: $3.5 \times 6 \text{ cm} = 21 \text{ cm}$

The dimensions of the enlargement are 31.5 cm by 21 cm.

Example 3 Drawing a Scale Diagram that Is an Enlargement

Draw a scale diagram of this metal bracket. Use a scale factor of 1.5.

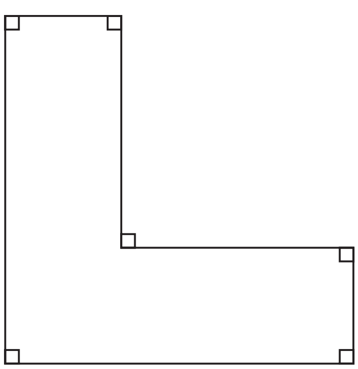


Solutions

Method 1

Use a photocopier. Write the scale factor 1.5 as a percent: 150%

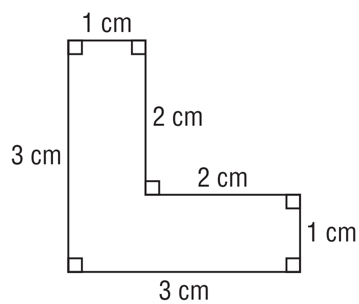
Set the zoom feature on the photocopier to 150%. Copy the diagram.



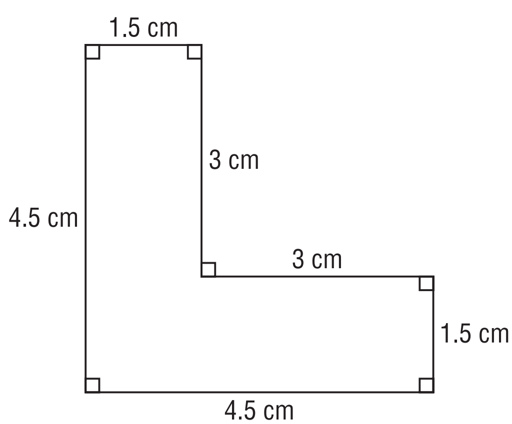
Method 2

Measure the length of each line segment in the given diagram.
Determine the length of each line segment in the scale diagram by multiplying each length on the original diagram by 1.5.

$1.5 \times 3 \text{ cm} = 4.5 \text{ cm}$
$1.5 \times 2 \text{ cm} = 3 \text{ cm}$
$1.5 \times 1 \text{ cm} = 1.5 \text{ cm}$



Use a ruler and a protractor to draw a scale diagram with the new lengths above.
The angles in the scale diagram match the angles in the given diagram.



Discuss the ideas

1. Explain what is meant by the term "scale factor" for a scale diagram.

2. When you calculate a scale factor, why is it important to have the same units for the lengths on the original diagram and the scale diagram?

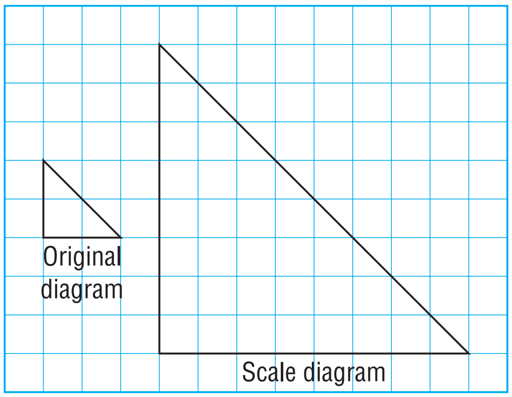
3. Suppose you are given two diagrams. How can you tell if one diagram is a scale drawing of the other diagram?

Practice

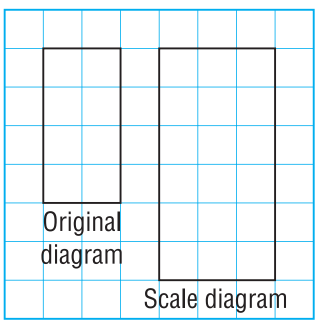
Check

4. Determine the scale factor for each scale diagram.

a)



b)



5. Scale diagrams of different squares are to be drawn. The side length of each original square and the scale factor are given. Determine the side length of each scale diagram.

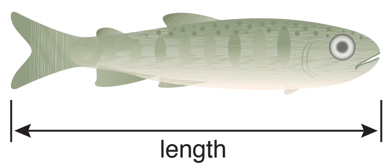
	Side length of original square	Scale factor
a)	12 cm	3
b)	82 mm	$\frac{5}{2}$
c)	1.55 cm	4.2
d)	45 mm	3.8
e)	0.8 cm	12.5

Apply

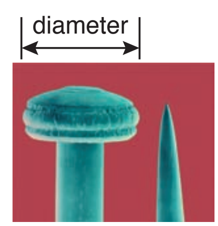
6. A photo of a surfboard has dimensions 17.5 cm by 12.5 cm. Enlargements are to be made with each scale factor below. Determine the dimensions of each enlargement. Round the answers to the nearest centimetre.

a) scale factor 12 **b)** scale factor 20
c) scale factor $\frac{7}{2}$ **d)** scale factor $\frac{17}{4}$

7. Here is a scale diagram of a salmon fry. The actual length of the salmon fry is 30 mm. Measure the length on the diagram to the nearest millimetre. Determine the scale factor for the scale diagram.



8. The head of a pin has diameter 2 mm. Determine the scale factor of this photo of the pinhead.



9. This view of the head of a bolt has the shape of a regular hexagon. Each angle is 120°. Use a protractor and ruler to draw a scale diagram of the bolt with scale factor 2.5.

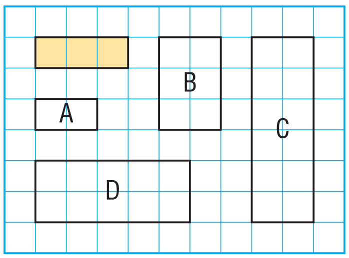
10. Draw your initials on 0.5-cm grid paper. Use different-sized grid paper to draw two different scale diagrams of your initials. For each scale diagram, state the scale factor.

11. **Assessment Focus** For each set of diagrams below, identify which of diagrams A, B, C, and D are scale diagrams of the shaded shape. For each scale diagram you identify:

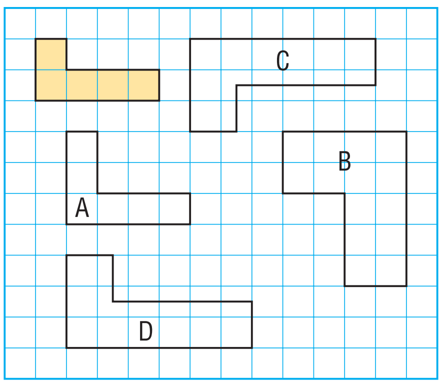
i) State the scale factor.

ii) Explain how it is a scale diagram.

a)



b)



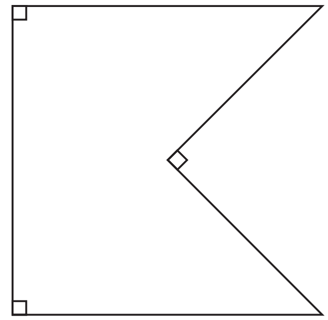
12. One frame of a film in a projector is about 50 mm high. The film is projected onto a giant screen. The image of the film frame is 16 m high.

a) What is the scale factor of this enlargement?

b) A penguin is 35 mm high on the film. How high is the penguin on the screen?

13. Look in a newspaper, magazine, or on the Internet. Find an example of a scale diagram that is an enlargement and has its scale factor given. What does the scale factor indicate about the original diagram or object?

14. Draw a scale diagram of the shape below with scale factor 2.5.



15. On a grid, draw ΔOAB with vertices O(0, 0), A(0, 3), and B(4, 0).

a) Draw a scale diagram of ΔOAB with scale factor 3 and one vertex at C(3, 3). Write the coordinates of the new vertices.

b) Is there more than one answer for part a? If your answer is no, explain why no other diagrams are possible. If your answer is yes, draw other possible scale diagrams.

Take It Further

16. One micron is one-millionth of a metre, or 1 m = 10^6 microns.

a) A human hair is about 200 microns wide. How wide is a scale drawing of a human hair with scale factor 400? Give your answer in as many different units as you can.

b) A computer chip is about 4 microns wide. A scale diagram of a computer chip is 5 cm wide. What is the scale factor?

Reflect

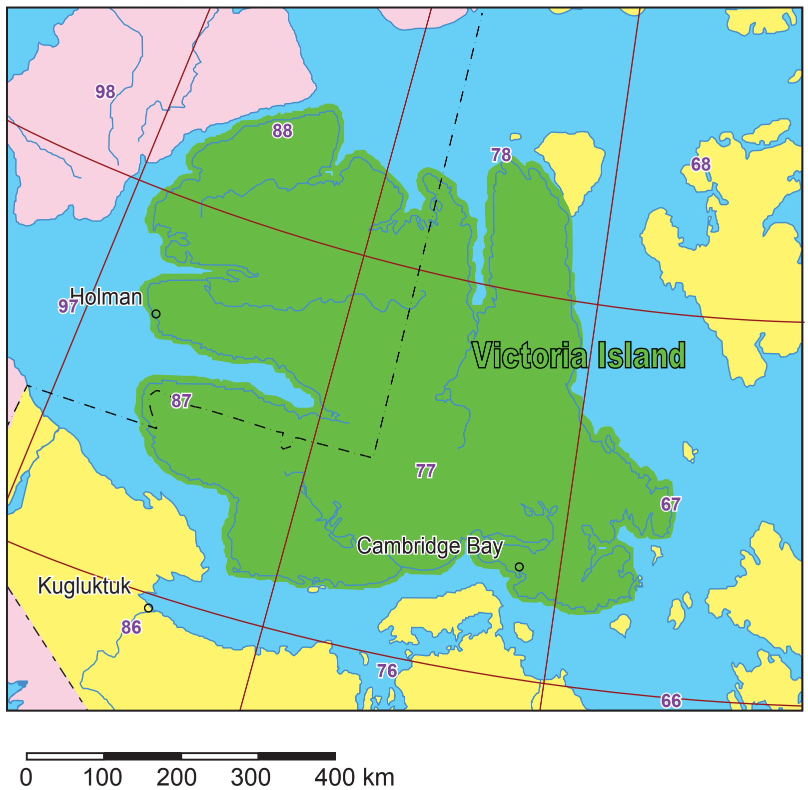
Suppose you are given a scale diagram. Why is it important to know the scale factor?

7.2 Scale Diagrams and Reductions

FOCUS

- Draw and interpret scale diagrams that represent reductions.

Here is a map of Victoria Island from the Internet.
What is the scale on the map? How is the scale used?



Investigate

2

You will need 2-cm grid paper and 0.5-cm grid paper.

➤ Trace your hand on the 2-cm grid paper. Copy this outline of your hand onto the 0.5-cm grid paper. Be as accurate as you can.

➤ On both drawings, measure and label the length of each finger.

For each finger, determine the fraction: $\dfrac{\text{Length on 0.5-cm grid paper}}{\text{Length on 2-cm grid paper}}$

Write each fraction as a decimal to the nearest hundredth.
What do you notice about the decimals?

Reflect & Share

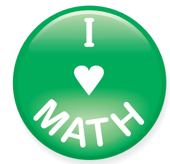
Compare your answers with those of another pair of classmates.
Are the numbers the same? Should they be the same? Explain.
How does this work relate to the scale diagrams of the previous lesson?

Connect

A scale diagram can be smaller than the original diagram. This type of scale diagram is called a *reduction*.

Here is a life-size drawing of a button and a scale diagram that is a reduction.

Original diagram



Scale diagram

We measure and compare corresponding lengths in the scale diagram and in the original diagram.

$$\frac{\text{Diameter of scale diagram}}{\text{Diameter of original diagram}} = \frac{2\text{ cm}}{3\text{ cm}} = \frac{2}{3}$$

$$\frac{\text{Height of heart on scale diagram}}{\text{Height of heart on original diagram}} = \frac{0.4\text{ cm}}{0.6\text{ cm}} = \frac{0.4}{0.6} = \frac{2}{3}$$

Write an equivalent fraction.

The fraction $\frac{\text{Length on scale diagram}}{\text{Length on original diagram}}$ is the scale factor of the scale diagram.

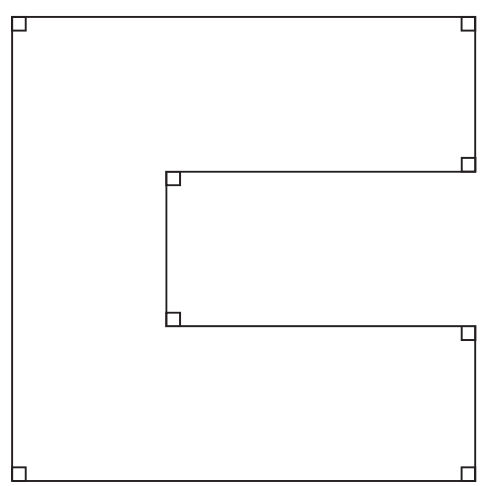
Pairs of corresponding lengths are proportional, and the scale factor is $\frac{2}{3}$.

The equation $\frac{\text{Length on scale diagram}}{\text{Length on original diagram}} = \frac{2}{3}$ is a proportion.

Each side of the reduction is shorter than the corresponding side on the original diagram, so the scale factor is less than 1.

Example 1 Drawing a Scale Diagram that Is a Reduction

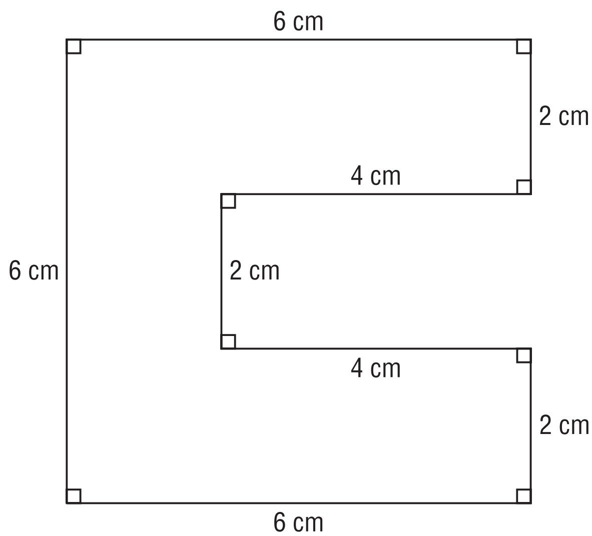
Draw a scale diagram of this octagon.
Use a scale factor of 0.25.



Solutions

Method 1

Measure the length of each line segment in the octagon.



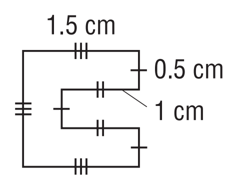
Determine the length of each line segment in the scale diagram by multiplying each length by 0.25.

$0.25 \times 2 \text{ cm} = 0.5 \text{ cm}$

$0.25 \times 4 \text{ cm} = 1 \text{ cm}$

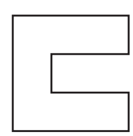
$0.25 \times 6 \text{ cm} = 1.5 \text{ cm}$

Use a ruler and protractor to draw a scale diagram with the new lengths above.
The angles in the scale diagram match the angles in the original diagram.



Method 2

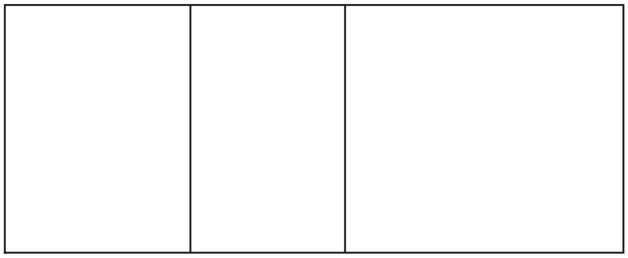
Use a photocopier.
Write the scale factor 0.25 as a percent: 25%
Set the zoom feature on the photocopier to 25%.
Copy the diagram.



A scale may be given as a ratio. For example, suppose the scale on a scale diagram of a house is 1:150. This means that 1 cm on the diagram represents 150 cm, or 1.5 m on the house.

Example 2 Using a Scale on a Scale Diagram to Determine Lengths

Here is a scale diagram of the top view of a truck.



Scale 1:50

The length of the truck is 4 m.

a) The front and back wheels of the truck are 3.85 m apart.
How far apart should the wheels be on the scale diagram?

b) What is the width of the truck?

A Solution

The scale is 1:50. This means that 1 cm on the diagram represents 50 cm on the truck.
So, the scale factor is $\frac{1}{50}$.

a) The front and back wheels of the truck are 3.85 m apart.
Each distance on the scale diagram is $\frac{1}{50}$ of its distance on the truck.
So, on the scale diagram, the distance between the wheels is:

$$\begin{aligned}\frac{1}{50} \times 3.85 \text{ m} &= \frac{3.85 \text{ m}}{50} \\ &= 0.077 \text{ m}\end{aligned}$$

Convert this length to centimetres: 0.077 m = 0.077 × 100 cm, or 7.7 cm
On the scale diagram, the wheels are 7.7 cm apart.

b) Measure the width of the truck on the scale diagram.
The width is 3.2 cm.
Each actual measure is 50 times as great as the measure on the scale diagram.
So, the actual width of the truck is: 50 × 3.2 cm = 160 cm
The truck is 160 cm wide; that is 1.6 m wide.

Discuss the ideas

1. What is a reduction? How is it like an enlargement?
How is it different?

2. What is a proportion? When can it be used to solve a problem involving reductions?

3. How can you tell whether a scale diagram is an enlargement or a reduction?

Practice

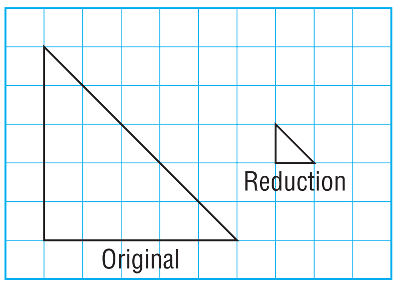
Check

4. Write each fraction in simplest form, then express it as a decimal.

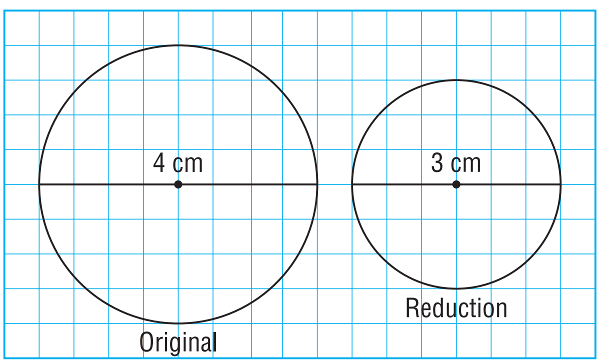
a) $\frac{25}{1000}$ b) $\frac{5}{125}$ c) $\frac{2}{1000}$ d) $\frac{3}{180}$

5. Determine the scale factor for each reduction as a fraction or a decimal.

a)



b)

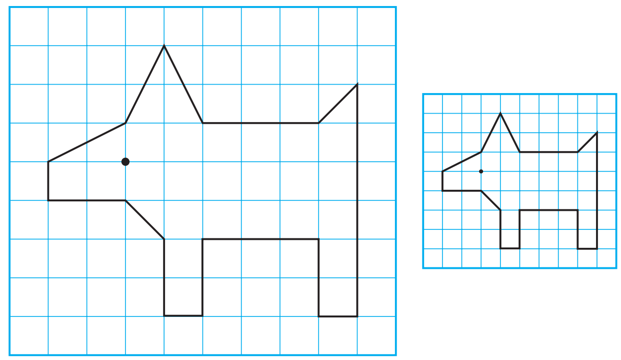


6. For each pair of circles, the original diameter and the diameter of the reduction are given. Determine each scale factor as a fraction or a decimal.

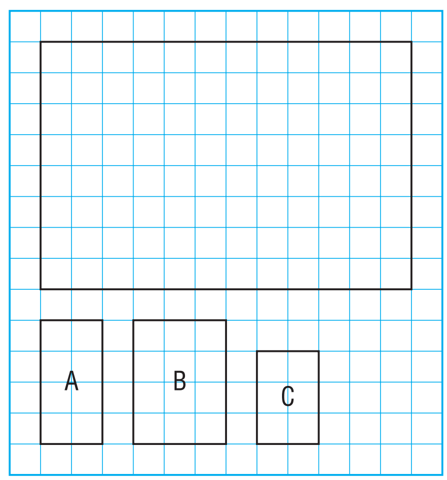
	Diameter of Actual Circle	Diameter of Reduction
a)	50 cm	30 cm
b)	30 cm	20 cm
c)	126 cm	34 cm
d)	5 m	2 cm
e)	4 km	300 m

Apply

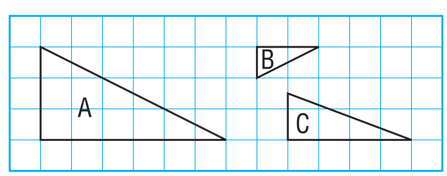
7. Here are two drawings of a dog. Determine the scale factor of the reduction as a fraction and as a decimal.



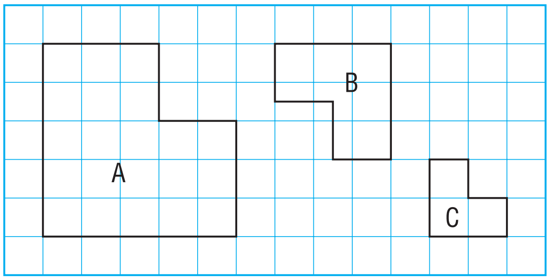
8. Which of rectangles A, B, and C is a reduction of the large rectangle? Justify your answer.



9. Which two polygons have pairs of corresponding lengths that are proportional? Identify the scale factor for the reduction.



10. Which two polygons have pairs of corresponding lengths that are proportional? Identify the scale factor for the reduction.

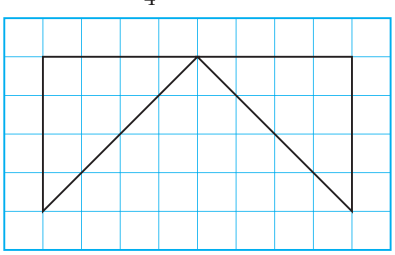


11. A reduction of each object is to be drawn with the given scale factor. Determine the corresponding length in centimetres on the scale diagram.

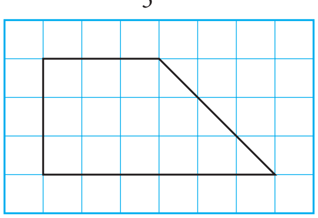
a) A desk has length 75 cm. The scale factor is $\frac{1}{3}$.
b) A bicycle has a wheel with diameter about 60 cm. The scale factor is $\frac{3}{50}$.
c) A surfboard has length 200 cm. The scale factor is 0.05.
d) A sailboat has length 8 m. The scale factor is 0.02.
e) A canyon has length 12 km. The scale factor is 0.000 04.

12. Copy each diagram on 1-cm grid paper. Draw a reduction of each diagram with the given scale factor.

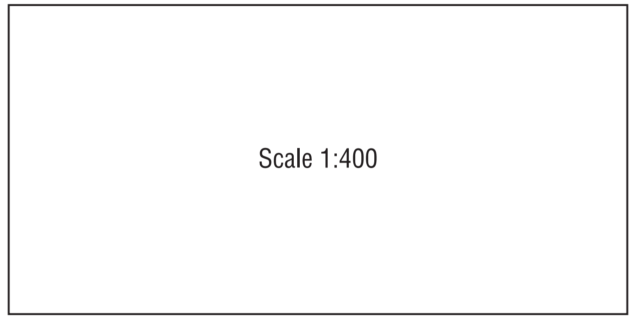
a) scale factor $\frac{3}{4}$



b) scale factor $\frac{2}{3}$



13. Here is a scale diagram of an outdoor hockey rink. The rink is 32 m long.



a) Each hockey net is 1.82 m long. Suppose you had to include the hockey nets on the scale diagram. How long would each hockey net be on the diagram?
b) What is the width of the rink?

14. A volleyball court measures approximately 18 m by 9 m. Make a scale drawing of the court using a scale factor of $\frac{1}{200}$. Show any calculations you made.

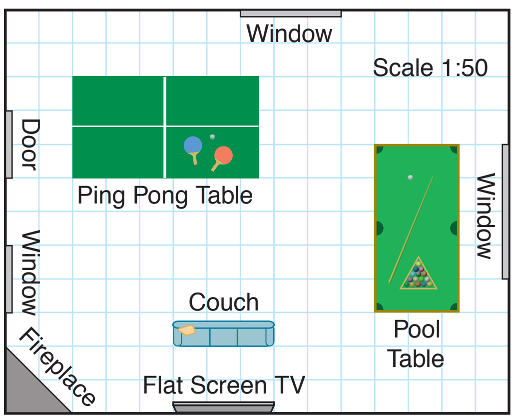
15. A lacrosse field measures 99 m by 54 m. Make a scale drawing of the field using a scale factor of 0.002. Show any calculations you made.

16. Your teacher will give you the dimensions of your classroom. Choose a scale factor and justify its choice. Draw a scale diagram of your classroom. Include as much detail as possible.

17. **Assessment Focus** Draw a scale diagram of any room in your home. Show as much detail as possible by including items in the room. Show any calculations you make and record the scale factor.

18. Look in a newspaper, magazine, or on the Internet. Find an example of a scale diagram that is a reduction and has its scale factor given. What does the scale factor indicate about the original diagram or object?

19. Ask your teacher for a scale diagram of the room shown below. The length of the room is 7.5 m.



a) Determine the scale factor.
b) What are the actual dimensions of:
  i) the ping pong table?
  ii) the pool table?
c) What is the actual size of the flat screen television?
d) Moulding is to be placed around the ceiling. It costs \$4.99/m. How much will the moulding cost?

20. A 747 jet airplane is about 70 m long. A plastic model of this plane is 28 cm long.
a) Determine the scale factor of the model.
b) On the model, the wingspan is 24 cm. What is the wingspan on the 747 plane?
c) On the model, the tail is 7.6 cm high. What is the height of the tail on the 747 plane?

Take It Further

21. The approximate diameter of each planet in our solar system is given below.
Earth: 12 760 km; Jupiter: 142 800 km;
Mars: 6790 km; Mercury: 4880 km;
Neptune: 49 500 km; Saturn: 120 600 km;
Uranus: 51 120 km; Venus: 12 100 km
Create a scale drawing that includes all the planets. Justify your choice of scale factor. Label each planet with its actual diameter.

Reflect

A scale factor is the ratio of a length on a scale diagram to the actual length.
When you know two of these three values, how can you determine the third value?
Include an example in each case.

Technology

Drawing Scale Diagrams

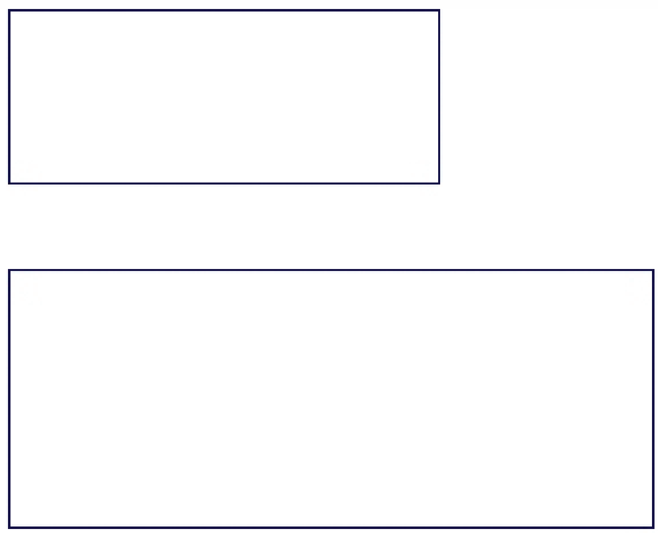
FOCUS

- Use different technologies to produce enlargements and reductions.

Geometry software can be used to enlarge or reduce a shape.
Use available geometry software.

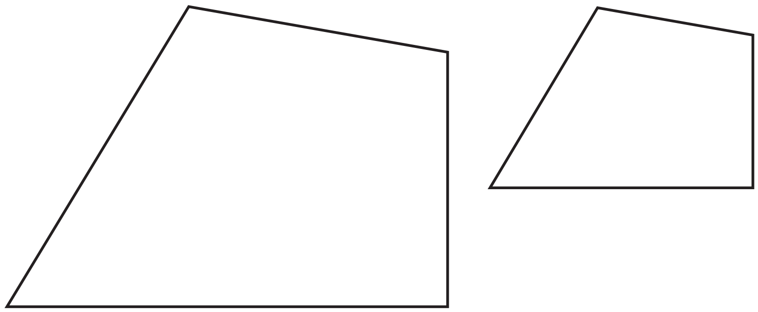
If you need help at any time, use the software's *Help* menu.

➤ Construct a rectangle. Select the rectangle.
Use the scale feature of the software to enlarge the rectangle.



The rectangle has been enlarged by a scale factor of 1.5, or 150%.

➤ Construct a quadrilateral. Select the quadrilateral.
Use the scale feature to reduce the quadrilateral.



The quadrilateral has been reduced by a scale factor of $\frac{3}{5}$, or 60%.

Check

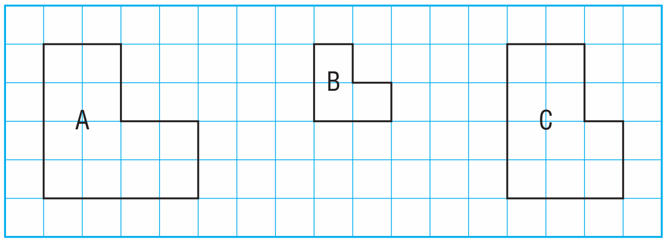
1. Construct a shape. Choose an enlargement scale factor, then enlarge your shape. Calculate the ratios of the corresponding sides of the enlargement and the original shape. What can you say about the ratios?
2. Construct a shape. Choose a reduction scale factor, then reduce your shape. Calculate the ratios of the corresponding sides of the reduction and the original shape. What can you say about the ratios?
3. Print the diagrams of the enlargement and reduction. Trade diagrams with a classmate. Identify the scale factor for each of your classmate's scale diagrams.
4. Try these other ways of enlarging and reducing a shape:
   - an overhead projector
   - a photocopier
   - a Draw tool in a software program

7.3 Similar Polygons

FOCUS

- Recognize and draw similar polygons, then use their properties to solve problems.

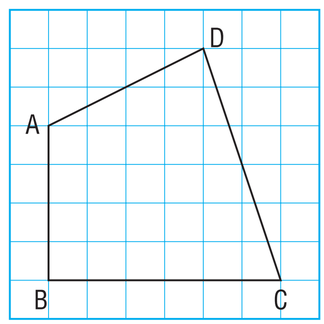
Which pair of polygons below show an enlargement or a reduction?
Explain your choice.



Investigate

2

You will need 0.5-cm grid paper, 2-cm grid paper, a ruler, and a protractor.



➤ Choose a scale factor. Draw an enlargement of quadrilateral ABCD.
Label the new quadrilateral A′B′C′D′.
Measure the side lengths to the nearest millimetre and the angles to the nearest degree.
Copy and complete this table:

Lengths of Sides (mm)	AB	A′B′	BC	B′C′	CD	C′D′	DA	D′A′
Measures of Angles (°)	∠A	∠A′	∠B	∠B′	∠C	∠C′	∠D	∠D′

➤ Choose a scale factor. Draw a reduction of quadrilateral ABCD.
Label the new quadrilateral A″B″C″D″. Copy and complete this table:

Lengths of Sides (mm)	AB	A″B″	BC	B″C″	CD	C″D″	DA	D″A″
Measures of Angles (°)	∠A	∠A″	∠B	∠B″	∠C	∠C″	∠D	∠D″

➤ Copy the table below. Use your results from the first 2 tables to complete this table.
Write the ratios of the lengths of the sides as decimals to the nearest hundredth.

$\frac{AB}{A'B'}$	$\frac{BC}{B'C'}$	$\frac{CD}{C'D'}$	$\frac{DA}{D'A'}$	$\frac{AB}{A''B''}$	$\frac{BC}{B''C''}$	$\frac{CD}{C''D''}$	$\frac{DA}{D''A''}$

➤ What do you notice about the measures of the matching angles?
What do you notice about the ratios of matching sides?

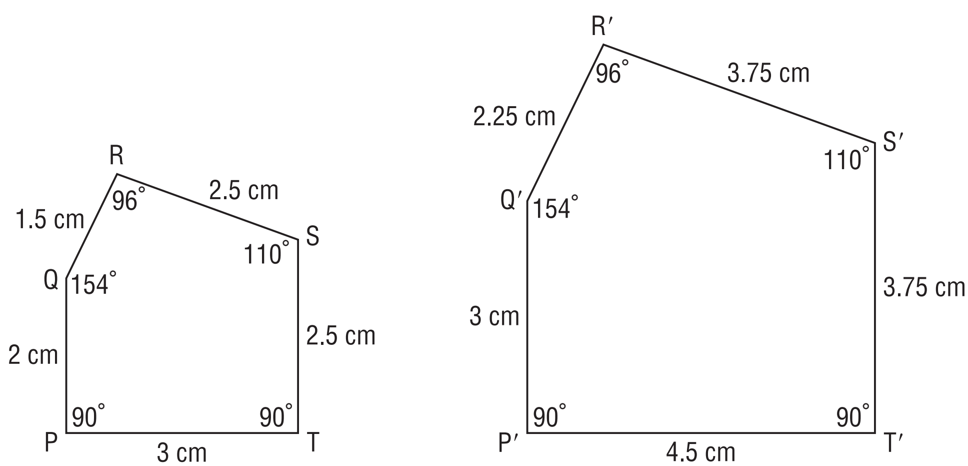
Reflect & Share

Compare your results with those of another pair of students.
Work together to draw two other quadrilaterals that have sides and angles related the same way as your quadrilaterals.
How does this work relate to scale drawings that show enlargements and reductions?

Connect

When one polygon is an enlargement or a reduction of another polygon, we say the polygons are **similar**. Similar polygons have the same shape, but not necessarily the same size.

Here are two similar pentagons.



Matching angles are **corresponding angles**.
Matching sides are **corresponding sides**.
We list the corresponding angles and the pairs of corresponding sides.

Corresponding Sides			Corresponding Angles	
PQ = 2 cm	P′Q′ = 3 cm	$\frac{P'Q'}{PQ} = \frac{3}{2} = 1.5$	∠P = 90°	∠P′ = 90°
QR = 1.5 cm	Q′R′ = 2.25 cm	$\frac{Q'R'}{QR} = \frac{2.25}{1.5} = 1.5$	∠Q = 154°	∠Q′ = 154°
RS = 2.5 cm	R′S′ = 3.75 cm	$\frac{R'S'}{RS} = \frac{3.75}{2.5} = 1.5$	∠R = 96°	∠R′ = 96°
ST = 2.5 cm	S′T′ = 3.75 cm	$\frac{S'T'}{ST} = \frac{3.75}{2.5} = 1.5$	∠S = 110°	∠S′ = 110°
TP = 3 cm	T′P′ = 4.5 cm	$\frac{T'P'}{TP} = \frac{4.5}{3} = 1.5$	∠T = 90°	∠T′ = 90°

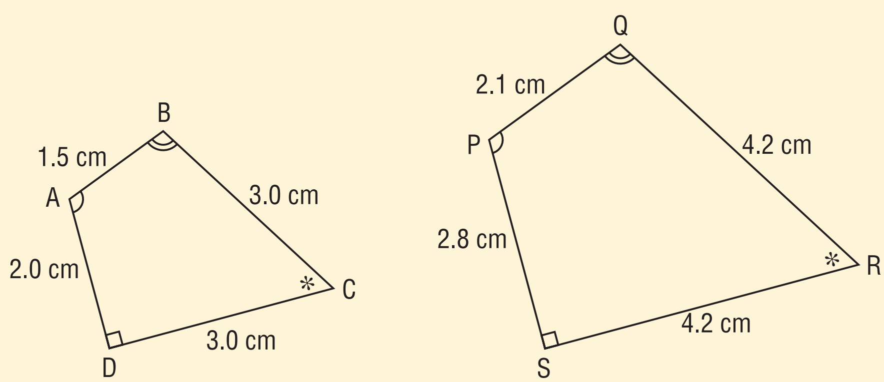
In similar polygons:
- pairs of corresponding sides have lengths in the same ratio; that is, the lengths are proportional, and
- the corresponding angles are equal

Pentagon P′Q′R′S′T′ is an enlargement of pentagon PQRST with a scale factor of $\frac{3}{2}$, or 1.5. Or, we can think of pentagon PQRST as a reduction of pentagon P′Q′R′S′T′ with a scale factor of $\frac{2}{3}$.
We say: pentagon PQRST is similar to P′Q′R′S′T′.
We write: pentagon PQRST ~ pentagon P′Q′R′S′T′

Properties of Similar Polygons

When two polygons are similar:
- their corresponding angles are equal, and
- their corresponding sides are proportional.

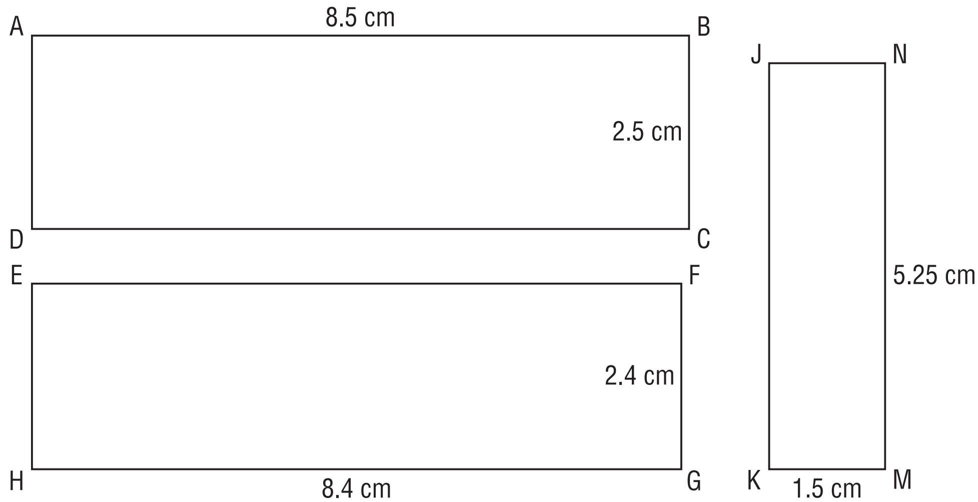
It is also true that if two polygons have these properties, then the polygons are similar.
Quadrilateral ABCD ~ quadrilateral PQRS



$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CD}{RS} = \frac{DA}{SP}$$

Example 1 Identifying Similar Polygons

Identify pairs of similar rectangles. Justify the answer.



A Solution

The measure of each angle in a rectangle is 90°.
So, for any two rectangles, their corresponding angles are equal.
For each pair of rectangles, determine the ratios of corresponding sides.
Since the opposite sides of a rectangle are equal, we only need to check the ratios of corresponding lengths and corresponding widths.

For rectangles ABCD and EFGH:

$\frac{AB}{EF} = \frac{8.5}{8.4}$ $\qquad$ $\frac{BC}{FG} = \frac{2.5}{2.4}$

$= 1.011\ldots$ $\qquad$ $= 1.041\overline{6}$

These numbers show that the corresponding sides are not proportional.
So, rectangles ABCD and EFGH are not similar.

For rectangles ABCD and JKMN:

$\frac{AB}{JK} = \frac{8.5}{5.25}$ $\qquad$ $\frac{BC}{KM} = \frac{2.5}{1.5}$

$= 1.619\ldots$ $\qquad$ $= 1.\overline{6}$

These numbers show that the corresponding sides are not proportional.
So, rectangles ABCD and JKMN are not similar.

For rectangles EFGH and JKMN:

$\frac{EF}{JK} = \frac{8.4}{5.25}$ $\qquad$ $\frac{FG}{KM} = \frac{2.4}{1.5}$

$= 1.6$ $\qquad$ $= 1.6$

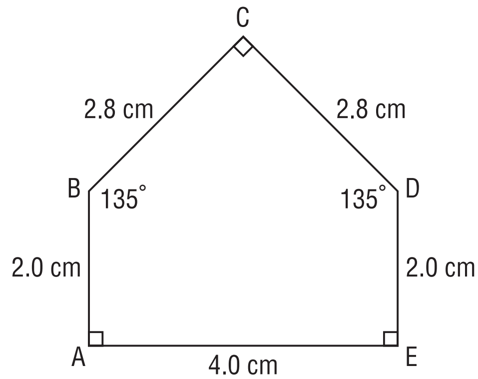
These numbers show that the corresponding sides are proportional.
So, rectangles EFGH and JKMN are similar.

Example 2 Drawing a Polygon Similar to a Given Polygon

a) Draw a larger pentagon that is similar to this pentagon.

b) Draw a smaller pentagon that is similar to this pentagon.

Explain why the pentagons are similar.



A Solution

a) Draw an enlargement. Choose a scale factor greater than 1, such as 2.
Let the similar pentagon be A′B′C′D′E′.
Multiply each side length of ABCDE by 2 to get the corresponding side lengths of A′B′C′D′E′.

$$\begin{aligned} A'B' &= 2 \times AB \\ &= 2 \times 2.0 \text{ cm} \\ &= 4.0 \text{ cm} \end{aligned} \qquad \begin{aligned} B'C' &= 2 \times BC \\ &= 2 \times 2.8 \text{ cm} \\ &= 5.6 \text{ cm} \end{aligned} \qquad \begin{aligned} E'A' &= 2 \times EA \\ &= 2 \times 4.0 \text{ cm} \\ &= 8.0 \text{ cm} \end{aligned}$$

Since DE = AB, then $D'E' = A'B' = 4.0$ cm

Since CD = BC, then $C'D' = B'C' = 5.6$ cm

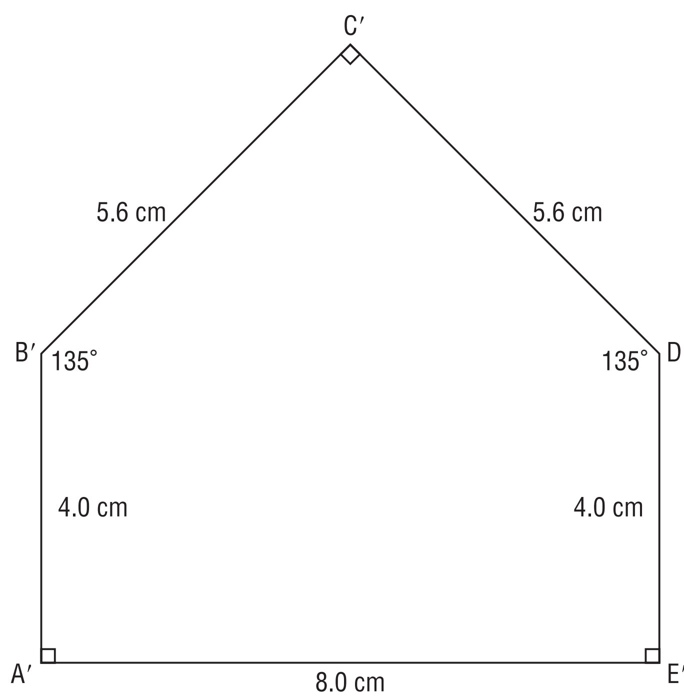
The corresponding angles are equal. So:

$$\begin{aligned} \angle A' &= \angle A \\ &= 90° \end{aligned} \qquad \begin{aligned} \angle B' &= \angle B \\ &= 135° \end{aligned}$$

$$\begin{aligned} \angle C' &= \angle C \\ &= 90° \end{aligned} \qquad \begin{aligned} \angle D' &= \angle D \\ &= 135° \end{aligned}$$

$$\begin{aligned} \angle E' &= \angle E \\ &= 90° \end{aligned}$$

Use a ruler and protractor to draw pentagon A′B′C′D′E′.
The pentagons are similar because corresponding angles are equal and corresponding sides are proportional. That is, the length of each side of the enlargement is 2 times the length of the corresponding side of the original pentagon.



b) Draw a reduction. Choose a scale factor that is less than 1, such as $\frac{1}{2}$.
Let the similar pentagon be A′B′C′D′E′.
Multiply each side length of ABCDE by $\frac{1}{2}$ to get the corresponding side lengths of A′B′C′D′E′.

$$A'B' = \frac{1}{2} \times AB \qquad B'C' = \frac{1}{2} \times BC \qquad E'A' = \frac{1}{2} \times EA$$
$$= \frac{1}{2} \times 2.0 \text{ cm} \qquad = \frac{1}{2} \times 2.8 \text{ cm} \qquad = \frac{1}{2} \times 4.0 \text{ cm}$$
$$= 1.0 \text{ cm} \qquad = 1.4 \text{ cm} \qquad = 2.0 \text{ cm}$$

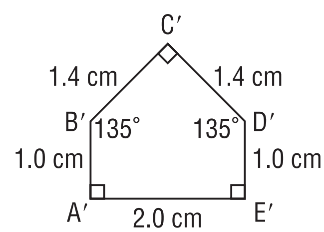
Since DE = AB, then D′E′ = A′B′ = 1.0 cm

Since CD = BC, then C′D′ = B′C′ = 1.4 cm

The corresponding angles are equal. So:

$$\angle A' = \angle A = 90° \qquad \angle B' = \angle B = 135° \qquad \angle C' = \angle C = 90° \qquad \angle D' = \angle D = 135° \qquad \angle E' = \angle E = 90°$$

Use a ruler and protractor to draw pentagon A′B′C′D′E′.
The pentagons are similar because corresponding angles are equal and corresponding sides are proportional.
That is, the length of each side of the reduction is $\frac{1}{2}$ the length of the corresponding side of the original pentagon.

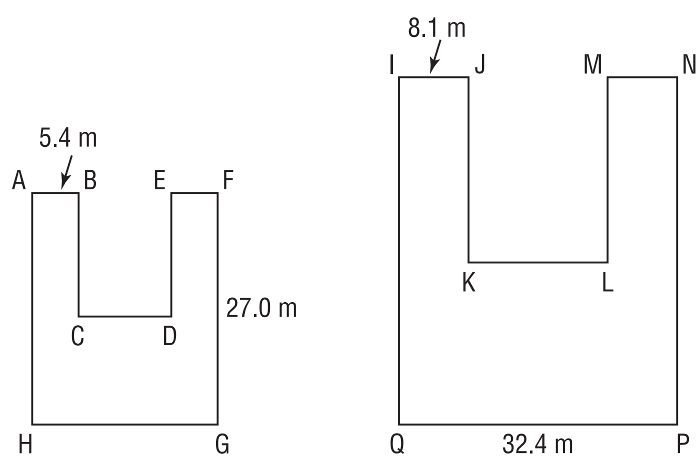


Example 3 Solving Problems Using the Properties of Similar Polygons

These two octagonal garden plots are similar.

a) Calculate the length of GH.

b) Calculate the length of NP.



A Solution

a) To calculate GH, consider polygon ABCDEFGH as a reduction of polygon IJKLMNPQ.

The scale factor of the reduction is the ratio of corresponding sides, such as:

$\frac{AB}{IJ} = \frac{5.4}{8.1}$

Write a ratio of corresponding sides that includes GH.

GH corresponds to PQ, so a ratio is $\frac{GH}{PQ}$.

Substitute: PQ = 32.4, then $\frac{GH}{PQ} = \frac{GH}{32.4}$

This ratio is equal to the scale factor.

Use the ratio and scale factor to write a proportion.

$\frac{GH}{32.4} = \frac{5.4}{8.1}$

Solve the proportion to determine GH. Multiply each side by 32.4.

$$32.4 \times \frac{GH}{32.4} = 32.4 \times \frac{5.4}{8.1}$$

$$GH = \frac{32.4 \times 5.4}{8.1}$$

$$GH = 21.6$$

GH is 21.6 m long.

b) To calculate NP, consider polygon IJKLMNPQ as an enlargement of polygon ABCDEFGH. The scale factor of the enlargement is the ratio of corresponding sides, such as: $\frac{IJ}{AB} = \frac{8.1}{5.4}$

Write a ratio of corresponding sides that includes NP.

NP corresponds to FG, so a ratio is $\frac{NP}{FG}$. This ratio is equal to the scale factor.

Substitute: FG = 27.0, then $\frac{NP}{FG} = \frac{NP}{27.0}$

Write a proportion.

$\frac{NP}{27.0} = \frac{8.1}{5.4}$

Solve the proportion to determine NP. Multiply each side by 27.0.

$$27.0 \times \frac{NP}{27.0} = 27.0 \times \frac{8.1}{5.4}$$

$$NP = \frac{27.0 \times 8.1}{5.4}$$

$$= 40.5$$

NP is 40.5 m long.

Discuss the ideas

1. How is drawing a similar polygon like drawing a scale diagram?

2. All rectangles have corresponding angles equal.

a) When would two rectangles be similar?

b) When would two rectangles *not* be similar?

3. How can you tell whether two polygons are similar?

Practice

Check

4. Calculate the side length, in units, in each proportion.

a) $\frac{AB}{8} = \frac{3}{2}$ **b)** $\frac{BC}{25} = \frac{12}{15}$

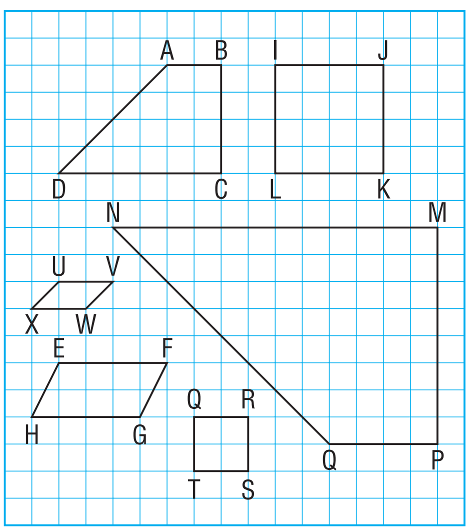
c) $\frac{CD}{4} = \frac{63}{28}$ **d)** $\frac{DE}{7} = \frac{24}{30}$

5. Calculate the value of the variable in each proportion.

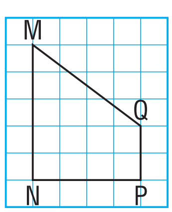
a) $\frac{x}{2.5} = \frac{7.5}{1.5}$ **b)** $\frac{y}{21.4} = \frac{23.7}{15.8}$

c) $\frac{z}{12.5} = \frac{0.8}{1.2}$ **d)** $\frac{a}{0.7} = \frac{1.8}{24}$

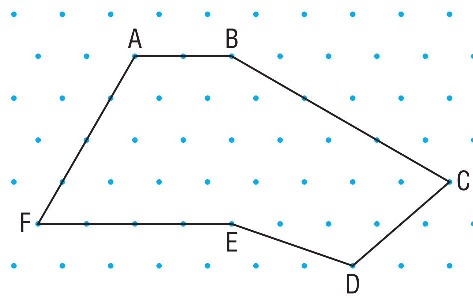
6. Identify similar quadrilaterals. List their corresponding sides and corresponding angles.



7. Use grid paper. Construct a quadrilateral similar to quadrilateral MNPQ.

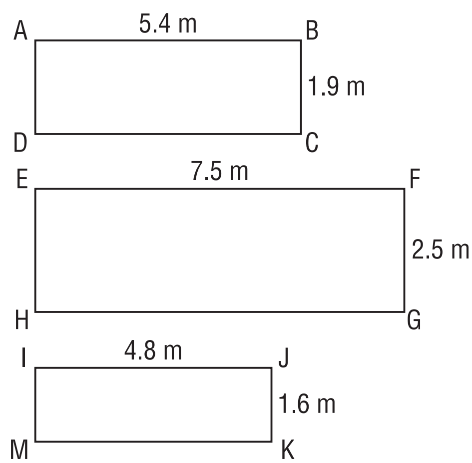


8. Use isometric dot paper. Construct a hexagon similar to hexagon ABCDEF.



Apply

9. Are any of these rectangles similar? Justify your answer.



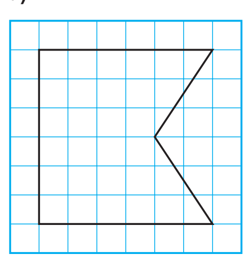
10. For each polygon below:

i) Draw a similar larger polygon.

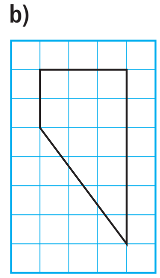
ii) Draw a similar smaller polygon.

Explain how you know the polygons are similar.

a)

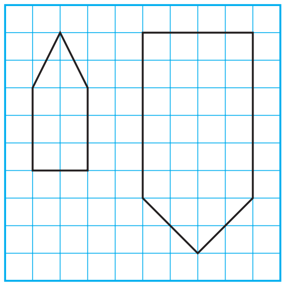


b)

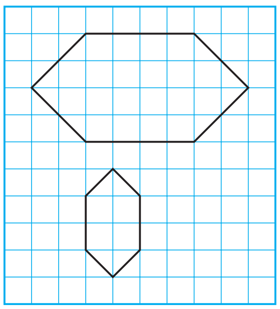


11. Are the polygons in each pair similar? Explain how you know.

a)



b)



12. **Assessment Focus** Use grid paper. Construct rectangles with these dimensions: 3 units by 4 units, 6 units by 8 units, 9 units by 12 units, and 12 units by 15 units

a) i) Which rectangle is not similar to the other rectangles? Explain your reasoning.

ii) Draw two different rectangles that are similar to this rectangle. Show your work.

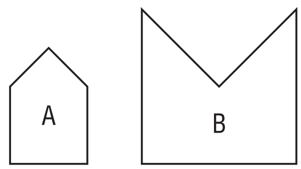
b) The diagonal of the smallest rectangle has length 5 units. Use proportions to calculate the lengths of the diagonals of the other two similar rectangles.

13. A rectangular door has height 200 cm and width 75 cm. It is similar to a door in a doll's house. The height of the doll's house door is 25 cm.

a) Sketch and label both doors.

b) Calculate the width of the doll's house door.

14. Each side of pentagon B is twice as long as a side of pentagon A.



Are the pentagons similar? Explain.

15. Use dot paper.

a) Draw two different:

i) equilateral triangles

ii) squares

iii) regular hexagons

b) Are all regular polygons of the same type similar? Justify your answer.

Take It Further

16. Are all circles similar? Justify your answer.

17. Draw two similar rectangles.

a) What is the ratio of their corresponding sides?

b) What is the ratio of their areas?

c) How are the ratios in parts a and b related?

d) Do you think the relationship in part c is true for all similar shapes? Justify your answer.

Reflect

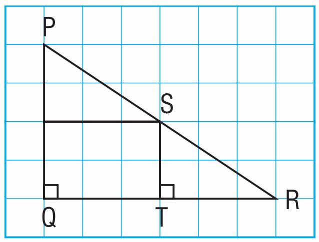
What is meant by the statement that two polygons are similar?
How would you check whether two polygons are similar?

7.4 Similar Triangles

FOCUS

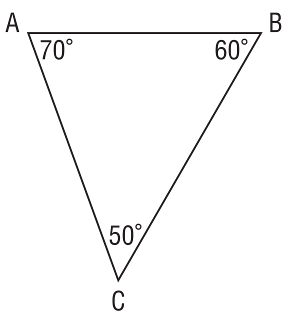
- Use the properties of similar triangles to solve problems.

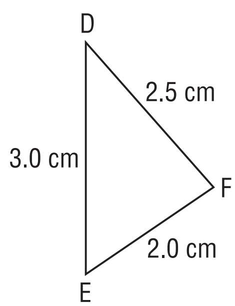
Identify two triangles in this diagram.
How could you find out if they are similar?

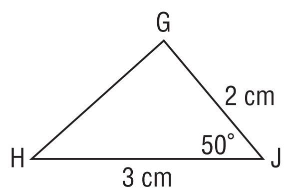


Investigate

You will need a ruler, compass, and protractor.
Each pair of students works with one of the three triangles below.







➤ Construct an enlargement of the triangle you chose. Label its vertices.
Construct a reduction of the triangle. Label its vertices.

➤ Measure and record the angles of all your triangles. What do you notice?

➤ Measure and record the ratios of the lengths of corresponding sides for:
- the original triangle and its enlargement
- the original triangle and its reduction

Write each ratio as a fraction, then as a decimal to the nearest tenth.
What do you notice about the decimals?

➤ What can you say about the triangles you worked with?

Reflect & Share

Compare your results with those of another group of classmates.
What do you need to know about two triangles to be able to identify whether they are similar?

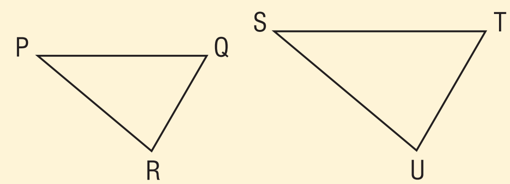
Connect

When two polygons are similar:
- the measures of corresponding angles must be equal *and*
- the ratios of the lengths of corresponding sides must be equal.

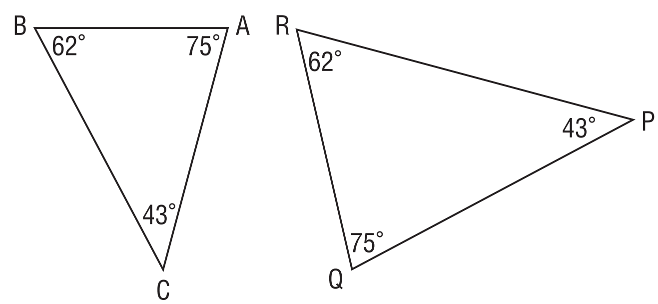
A triangle is a special polygon. When we check whether two triangles are similar:
- the measures of corresponding angles must be equal; *or*
- the ratios of the lengths of corresponding sides must be equal

Properties of Similar Triangles

To identify that ΔPQR and ΔSTU are similar, we only need to know that:
- $\angle P = \angle S$ and $\angle Q = \angle T$ and $\angle R = \angle U$; *or*
- $\frac{PQ}{ST} = \frac{QR}{TU} = \frac{PR}{SU}$



These triangles are similar because:
$\angle A = \angle Q = 75°$
$\angle B = \angle R = 62°$
$\angle C = \angle P = 43°$
When we name two similar triangles, we order the letters to match the corresponding angles.
We write: $\Delta ABC \sim \Delta QRP$
Then we can identify corresponding sides:
AB corresponds to QR.
BC corresponds to RP.
AC corresponds to QP.



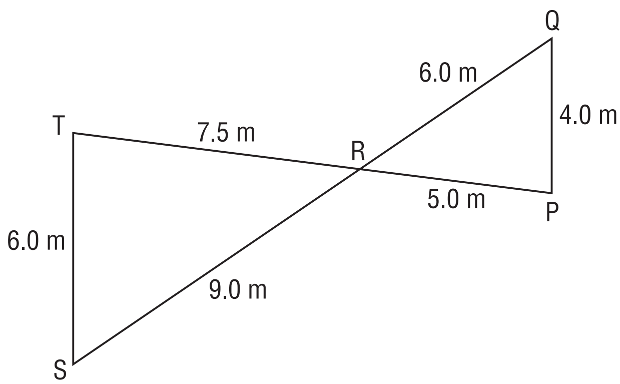
Math Link

Your World

Satellite imagery consists of photographs of Earth taken from space. The images are reductions of regions on Earth. The quality of an image depends upon the instrument used to obtain it and on the altitude of the satellite. The Landsat 7 satellite can create images of objects as small as 10 cm long.

Example 1 Using Corresponding Sides to Name Similar Triangles

Identify the similar triangles.
Justify your answer.



A Solution

Since we know the side lengths of the triangles,
we identify the corresponding sides.
In ΔPQR, from shortest to longest: PQ, PR, QR
In ΔSTR, from shortest to longest: ST, TR, RS
Find out if the corresponding sides are proportional.

$$\frac{ST}{PQ} = \frac{6.0}{4.0} = 1.5$$

$$\frac{TR}{PR} = \frac{7.5}{5.0} = 1.5$$

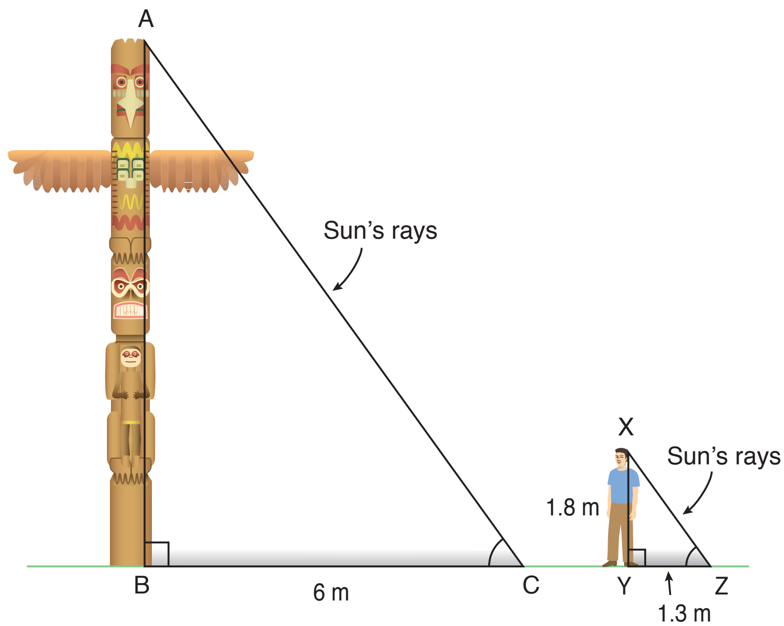
$$\frac{RS}{QR} = \frac{9.0}{6.0} = 1.5$$

Since the corresponding sides are proportional, the triangles are similar.
P and T are the vertices where the two shorter sides in each triangle meet,
so ∠P corresponds to ∠T.
Similarly, ∠Q corresponds to ∠S and ∠TRS corresponds to ∠QRP.
So, ΔPQR ~ ΔTSR

In *Example 1*, we can say that ΔTSR is an enlargement of ΔPQR
with a scale factor of 1.5.
Or, since $1.5 = \frac{3}{2}$, we can also say that ΔPQR is a reduction of ΔTSR
with a scale factor of $\frac{2}{3}$.
We can use the properties of similar triangles to solve problems
that involve scale diagrams.
These problems involve lengths that cannot be measured directly.

Example 2 Using Similar Triangles to Determine a Length

At a certain time of day, a person who is 1.8 m tall has a shadow 1.3 m long. At the same time, the shadow of a totem pole is 6 m long. The sun's rays intersect the ground at equal angles. How tall is the totem pole, to the nearest tenth of a metre?



A Solution

The sun's rays form two triangles with the totem pole, the person, and their shadows.
If we can show the triangles are similar, we can use a proportion to determine the height of the totem pole.
Assume both the totem pole and the person are perpendicular to the ground, so:
$\angle B = \angle Y = 90°$
The sun's rays make equal angles with the ground, so: $\angle C = \angle Z$
Since two pairs of corresponding angles are equal, the angles in the third pair must also be equal because the sum of the angles in each triangle is 180°.
So, $\angle A = \angle X$
Since 3 pairs of corresponding angles are equal, $\Delta ABC \sim \Delta XYZ$
So, ΔABC is an enlargement of ΔXYZ with a scale factor of $\frac{6}{1.3}$.
Write a proportion that includes the unknown height of the totem pole, AB.

$$\frac{AB}{XY} = \frac{6}{1.3}$$ Substitute XY = 1.8.

$$\frac{AB}{1.8} = \frac{6}{1.3}$$ To solve for AB, multiply each side by 1.8.

$$1.8 \times \frac{AB}{1.8} = \frac{6}{1.3} \times 1.8$$

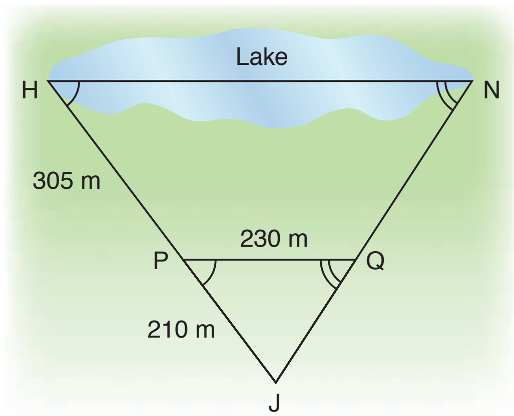
$$AB = \frac{6 \times 1.8}{1.3}$$

$$\doteq 8.308$$

The height of the totem pole is about 8.3 m.

Example 3 Using Overlapping Similar Triangles to Determine a Length

A surveyor wants to determine the width of a lake at two points on opposite sides of the lake. She measures distances and angles on land, then sketches this diagram. How can the surveyor determine the length HN to the nearest metre?



A Solution

Identify the two triangles, then draw them separately.

Consider ΔHNJ and ΔPQJ. From the diagram:

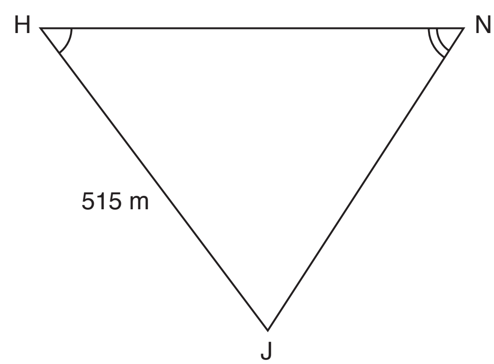
$\angle NHJ = \angle QPJ$

$\angle HNJ = \angle PQJ$

$\angle J$ is the common angle to both triangles.

Since 3 pairs of corresponding angles are equal,

$\Delta HNJ \sim \Delta PQJ$

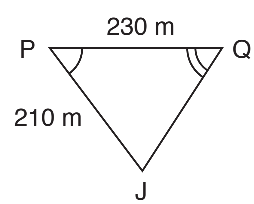


Two corresponding sides are:

$HJ = 305 \text{ m} + 210 \text{ m}$ and $PJ = 210 \text{ m}$

$= 515 \text{ m}$

So, ΔHNJ is an enlargement of ΔPQJ with a scale factor of $\frac{515}{210}$.



Write a proportion that includes the unknown length HN.

$$\frac{HN}{PQ} = \frac{515}{210} \qquad \text{Substitute } PQ = 230.$$

$$\frac{HN}{230} = \frac{515}{210} \qquad \text{To solve for HN, multiply each side by 230.}$$

$$230 \times \frac{HN}{230} = \frac{515}{210} \times 230$$

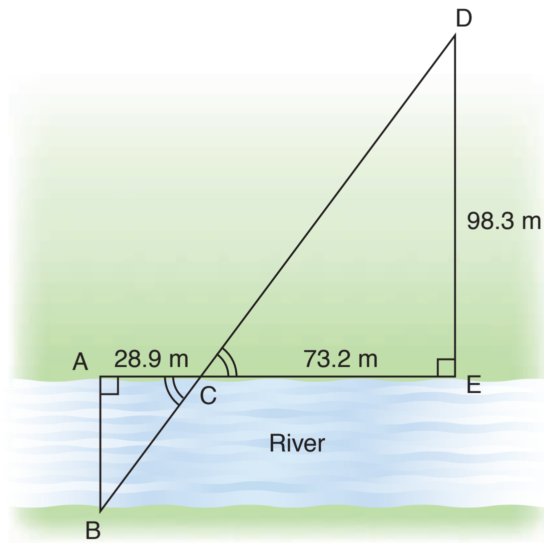
$$HN = \frac{515 \times 230}{210}$$

$$\doteq 564.0476$$

The width of the lake, HN, is about 564 m.

Example 4 Using Triangles Meeting at a Vertex to Determine a Length

A surveyor used this scale diagram to determine the width of a river. The measurements he made and the equal angles are shown. What is the width, AB, to the nearest tenth of a metre?



A Solution

Consider ΔABC and ΔEDC.
From the diagram:
∠A = ∠E
∠B = ∠D
∠ACB = ∠ECD
Since 3 pairs of corresponding angles are equal, ΔABC ~ ΔEDC
Two corresponding sides are:
AC = 28.9 m and EC = 73.2 m
So, ΔABC is a reduction of ΔEDC with a scale factor of $\frac{28.9}{73.2}$.

Write a proportion that includes the unknown length AB.

$$\frac{AB}{ED} = \frac{28.9}{73.2}$$ Substitute ED = 98.3.

$$\frac{AB}{98.3} = \frac{28.9}{73.2}$$ To solve for AB, multiply each side by 98.3.

$$98.3 \times \frac{AB}{98.3} = \frac{28.9}{73.2} \times 98.3$$

$$AB = \frac{28.9 \times 98.3}{73.2}$$

$$\doteq 38.8097$$

The width of the river, AB, is about 38.8 m.

Discuss the ideas

1. How can you tell that two triangles are similar?

2. When two triangles are similar, how do you identify the corresponding sides?

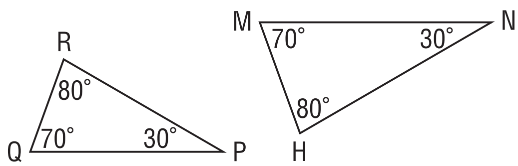
3. Suppose you know that two triangles are similar. How do you write the proportion to determine the length of an unknown side?

Practice

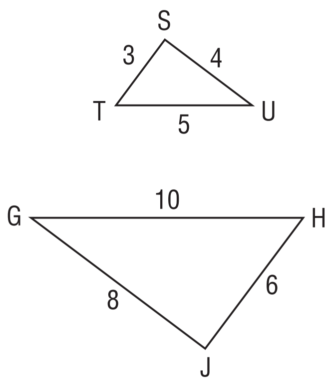
Check

4. Which triangles in each pair are similar? How do you know?

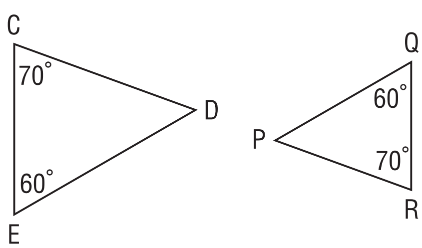
a)



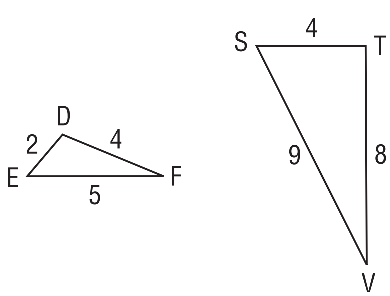
b)



c)

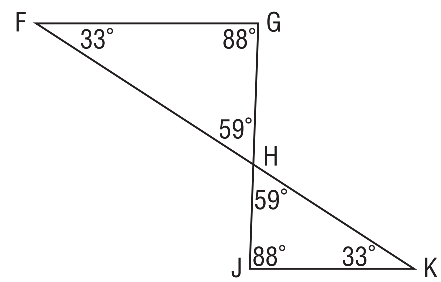


d)

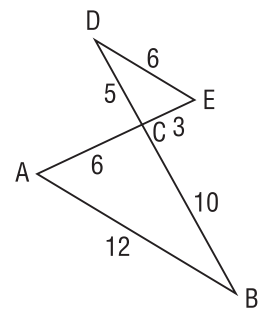


5. In each diagram, identify two similar triangles. Explain why they are similar.

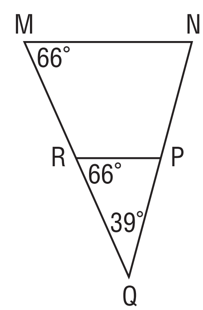
a)



b)



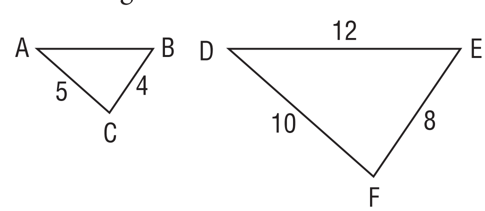
c)



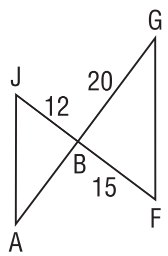
Apply

6. Determine the length of AB in each pair of similar triangles.

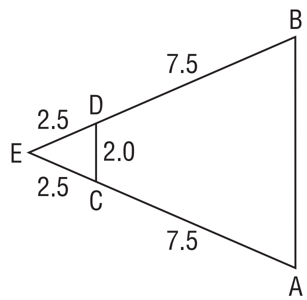
a)



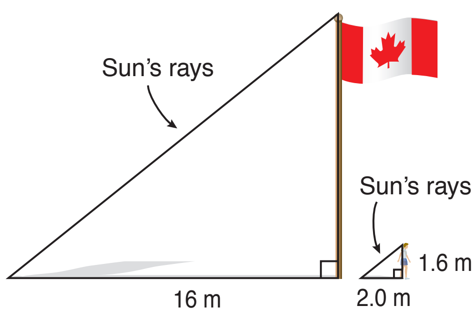
b)



c)

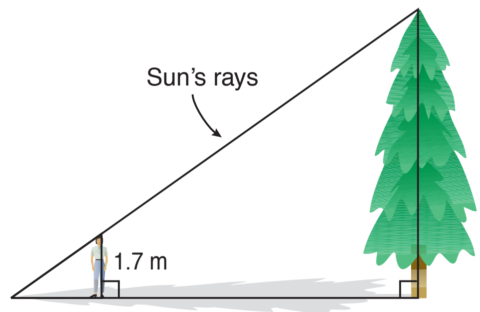


7. Jaquie is 1.6 m tall. When her shadow is 2.0 m long, the shadow of the school's flagpole is 16 m long. How tall is the flagpole, to the nearest tenth of a metre?



8. **Assessment Focus** Work with a partner. Use the method described in question 7. Choose an object whose height you cannot measure directly.
   a) Draw a labelled diagram.
   b) Indicate which triangles are similar.
   c) Determine the height of the object. Show your work.

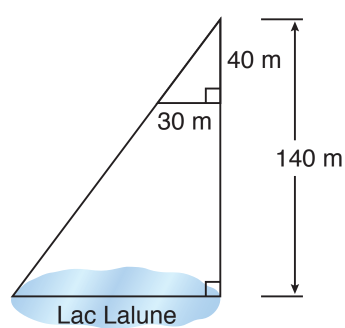
9. Tina wants to estimate the heights of two trees. For each tree, she stands so that one end of her shadow coincides with one end of the shadow of the tree. Tina's friend measures the lengths of her shadow and the tree's shadow. Tina is 1.7 m tall.



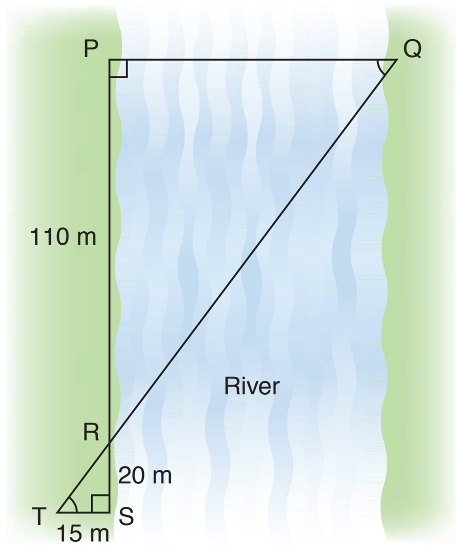
   a) Tina's shadow is 2.4 m and the first tree's shadow is 10.8 m. What is the height of the tree?
   b) Tina's shadow is 0.8 m and the second tree's shadow is 12.8 m. What is the height of the tree?

10. When the shadow of a building is 16 m long, a 4-m fence post casts a shadow 3 m long.
   a) Sketch a diagram.
   b) How tall is the building?

11. This scale diagram shows the measurements a surveyor made to determine the length of Lac Lalune. What is this length? How do you know?

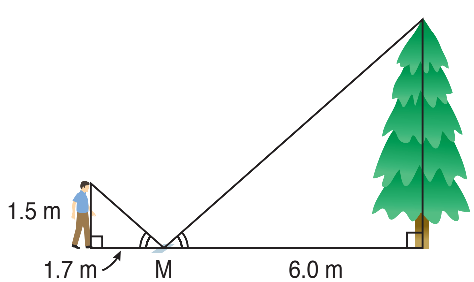


12. To help calculate the distance PQ across a river, Emil drew the diagram below based on measurements he made. What is the distance across the river?

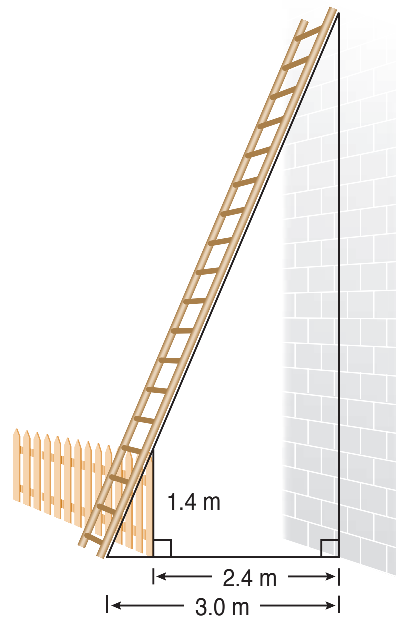


Take It Further

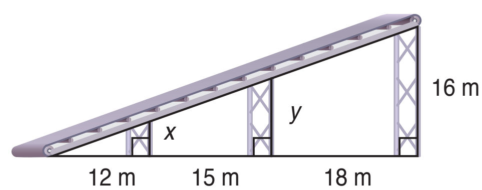
13. Phillipe places a mirror M on the ground 6.0 m from a tree. When he is 1.7 m from the mirror, he can see the top of the tree in the mirror. His eyes are 1.5 m above the ground. The diagram below shows the equal angles. How can you use similar triangles to determine the height of the tree to the nearest tenth of a metre?



14. The foot of a ladder is 3 m from the base of a wall. The ladder just touches the top of a 1.4-m fence that is 2.4 m from the wall. How high up the wall does the ladder reach? How do you know?



15. In the diagram below, how high are the two supports x and y for the conveyor belt?



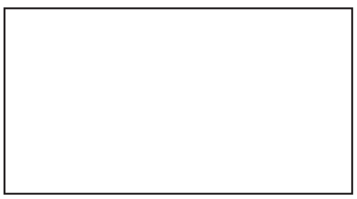
Reflect

How do the properties of similar triangles help you to determine distances that cannot be measured directly? Include an example in your explanation.

Mid-Unit Review

7.1 **1.** A photo of a gymnast is to be enlarged. The dimensions of the photo are 15 cm by 10 cm. What are the dimensions of the enlargement with a scale factor of $\frac{7}{5}$?

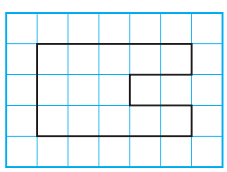
2. A computer chip has dimensions 15 mm by 8 mm. Here is a scale drawing of the chip.



a) Determine the scale factor of the diagram.

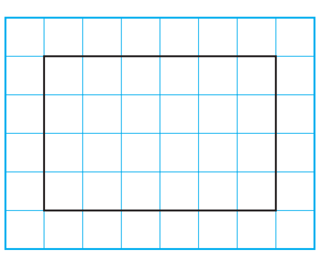
b) Draw a scale diagram of the chip with a scale factor of 8.

7.2 **3. a)** Copy this polygon on 1-cm grid paper.

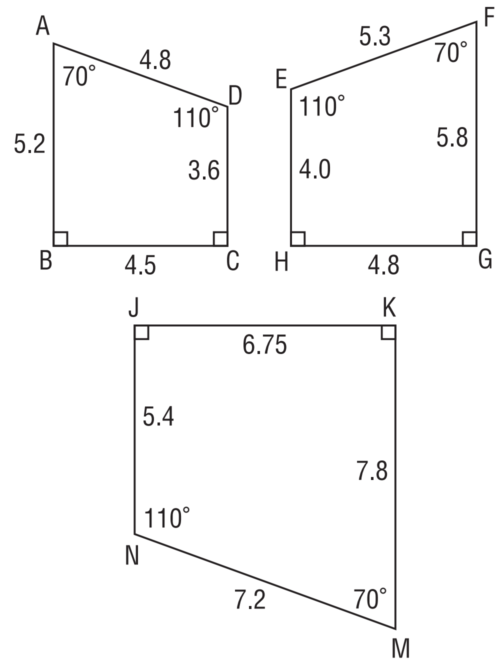


b) Draw a scale diagram of the polygon with a scale factor of $\frac{3}{5}$. Show any calculations you made.

4. This top view of a swimming pool is drawn on 0.5-cm grid paper. The dimensions of the pool are 60 m by 40 m. Determine the scale factor of the reduction as a fraction or a decimal.



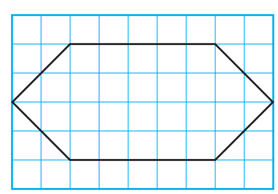
7.3 **5.** These quadrilaterals have corresponding angles equal.



a) Are any of these quadrilaterals similar? Justify your answer.

b) Choose one quadrilateral. Draw a similar quadrilateral. How do you know the quadrilaterals are similar?

6. A window has the shape of a hexagon.



Draw a hexagon that is similar to this hexagon. Explain how you know the hexagons are similar.

7.4 **7.** A tree casts a shadow 8 m long. At the same time a 2-m wall casts a shadow 1.6 m long.

a) Sketch a diagram.

b) What is the height of the tree?

7.5 Reflections and Line Symmetry

FOCUS

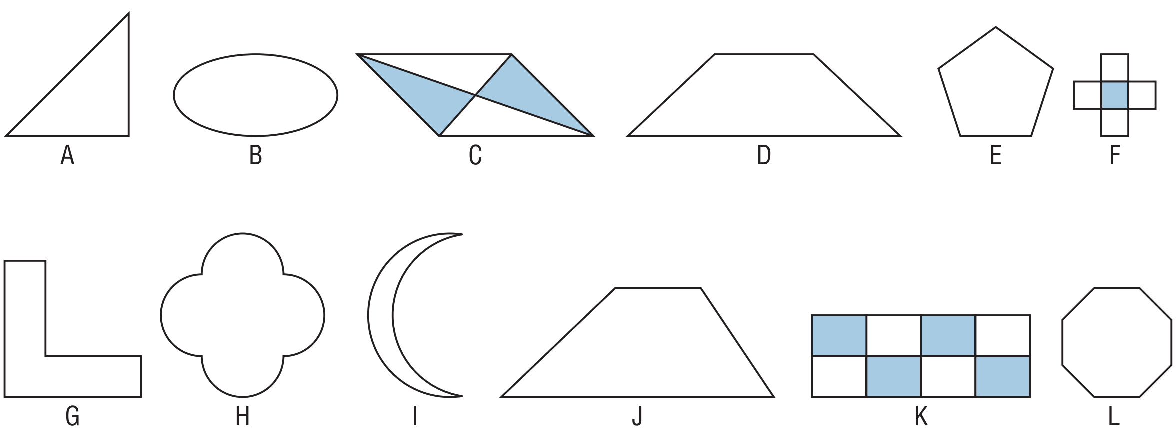
- Draw and classify shapes with line symmetry.

How can you use this photograph to show what you know about line symmetry?

Investigate

2

Your teacher will give you a large copy of the shapes below.



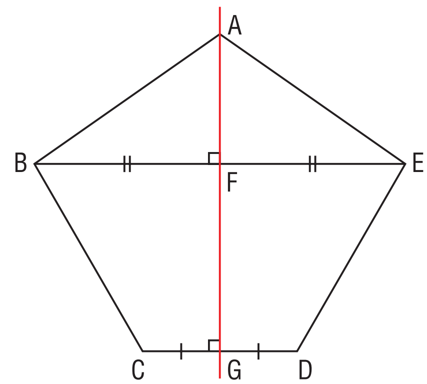
Which shapes have the same number of lines of symmetry?
Sort the shapes according to the number of lines of symmetry they have.
Which shapes do not have line symmetry? How can you tell?

Reflect & Share

Share your sorting with another pair of students.
Compare strategies for identifying the lines of symmetry.

Connect

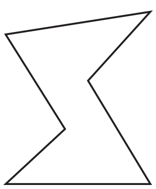
The pentagon ABCDE has one line of symmetry AG, because AG divides the pentagon ABCDE into two congruent parts: polygon ABCG is congruent to polygon AEDG.



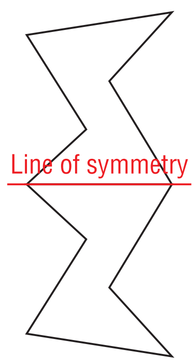
Also, each point on one side of the line of symmetry has a corresponding point on the other side of the line. These two points are the same distance, or *equidistant* from the line of symmetry:
points B and E correspond, BF = FE, and BE ⊥ AG.

A line of symmetry is also called a *line of reflection*. If a mirror is placed along one side of a shape, the reflection image and the original shape together form one larger shape. The line of reflection is a line of symmetry of this larger shape.

Original shape

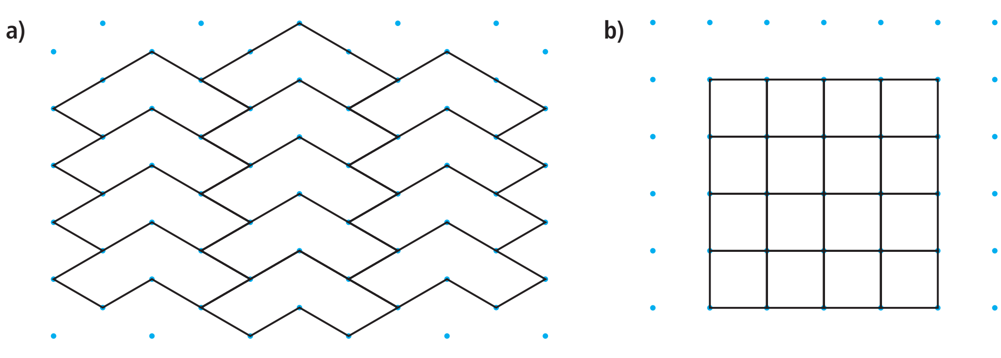


Original shape and its reflection image



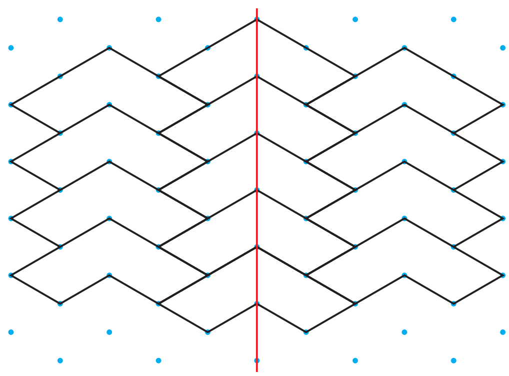
Example 1 Identifying Lines of Symmetry in Tessellations

Identify the lines of symmetry in each tessellation.

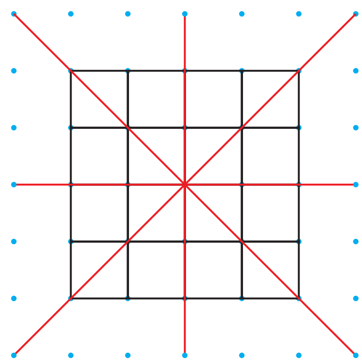


A Solution

a) The red line is the line of symmetry for this tessellation. Each point on one side of the line has a corresponding point on the other side. The pattern on one side of the line of symmetry is a mirror image of the pattern on the other side.



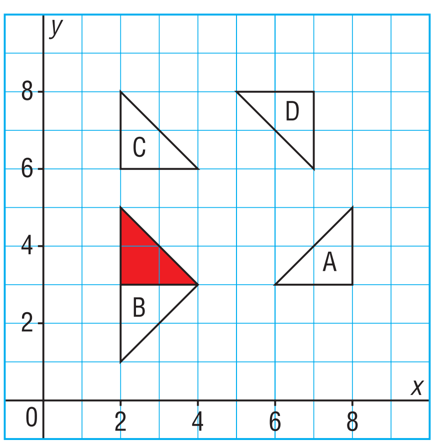
b) This tessellation has 4 lines of symmetry. For each line, a point on one side of the line has a matching point on the other side. And, the pattern on one side of the line is a mirror image of the pattern on the other side.



Two shapes may be related by a line of reflection.

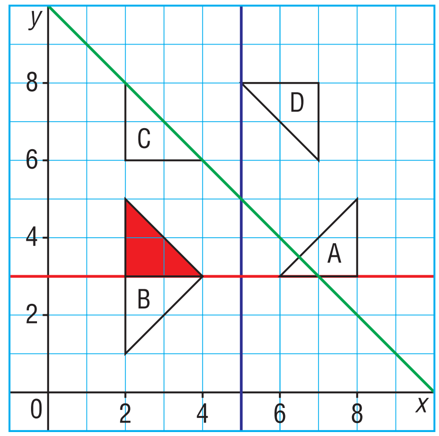
Example 2 Identifying Shapes Related by a Line of Reflection

Identify the triangles that are related to the red triangle by a line of reflection. Describe the position of each line of symmetry.



A Solution

Triangle A is the reflection image of the red triangle in the blue line through 5 on the *x*-axis.
Triangle B is the reflection image of the red triangle in the red line through 3 on the *y*-axis.
Triangle C is not a reflection image of the red triangle.
Triangle D is the reflection image of the red triangle in the green line through the points (9, 1) and (1, 9).



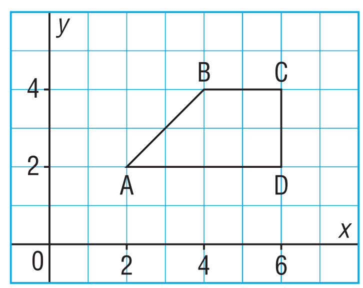
We can use a coordinate grid to draw shapes and their reflection images.

Example 3 Completing a Shape Given its Line of Symmetry

Quadrilateral ABCD is part of a larger shape.

- Draw the image of ABCD after each reflection below.
- Write the coordinates of the larger shape formed by ABCD and its image.
- Describe the larger shape and its symmetry.

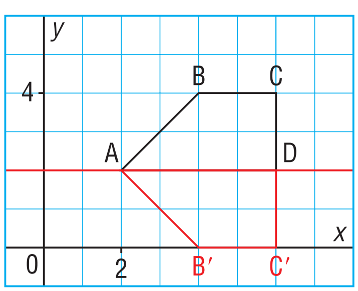
a) a reflection in the horizontal line through 2 on the *y*-axis
b) a reflection in the vertical line through 6 on the *x*-axis
c) a reflection in an oblique line through (0, 0) and (6, 6)



A Solution

The red line is the line of reflection. Each image point is the same distance from this line as the corresponding original point.

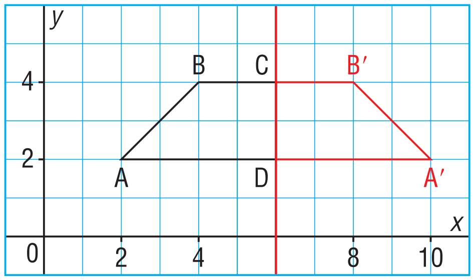
a)



Point	Image
A(2, 2)	A(2, 2)
B(4, 4)	B′(4, 0)
C(6, 4)	C′(6, 0)
D(6, 2)	D(6, 2)

The larger shape ABCC′B′ has coordinates: A(2, 2), B(4, 4), C(6, 4), C′(6, 0), B′(4, 0)
This shape is a pentagon with line symmetry. The line of symmetry is the red line.

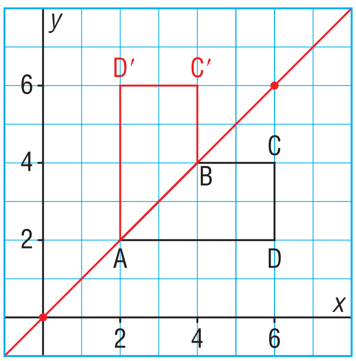
b)



Point	Image
A(2, 2)	A′(10, 2)
B(4, 4)	B′(8, 4)
C(6, 4)	C(6, 4)
D(6, 2)	D(6, 2)

The larger shape ABB′A′ has coordinates: A(2, 2), B(4, 4), B′(8, 4), A′(10, 2)
This shape is an isosceles trapezoid with line symmetry. The line of symmetry is the red line.

c)



Point	Image
A(2, 2)	A(2, 2)
B(4, 4)	B(4, 4)
C(6, 4)	C′(4, 6)
D(6, 2)	D′(2, 6)

The larger shape AD′C′BCD has coordinates: A(2, 2), D′(2, 6), C′(4, 6), B(4, 4), C(6, 4), D(6, 2)
This shape is a concave hexagon with line symmetry. The line of symmetry is the red line.

Discuss the ideas

1. How do you identify whether a shape has a line of symmetry?

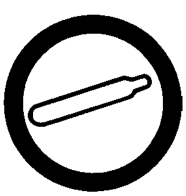
2. How are a line of reflection and a line of symmetry related?

Practice

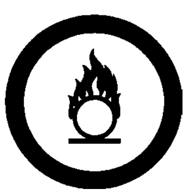
Check

3. You may have seen these hazardous substance warning symbols in the science lab. Which symbols have line symmetry? How many lines of symmetry?

a)

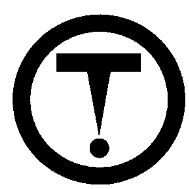


b)



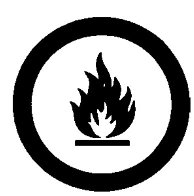
c)

d)



e)

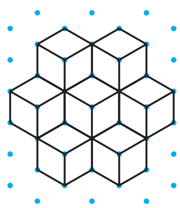
f)



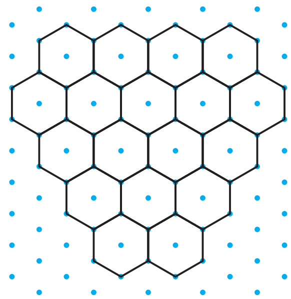
Apply

4. Identify the lines of symmetry in each tessellation.

a)

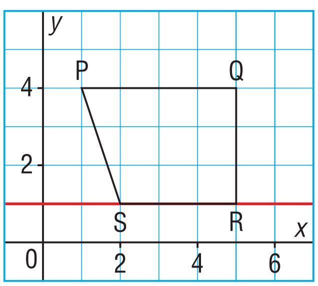


b)

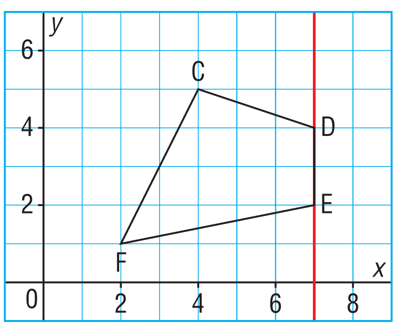


5. Copy each polygon on grid paper. It is one-half of a shape. Use the red line as a line of symmetry to complete the shape by drawing its other half. Label the shape with the coordinates of its vertices.

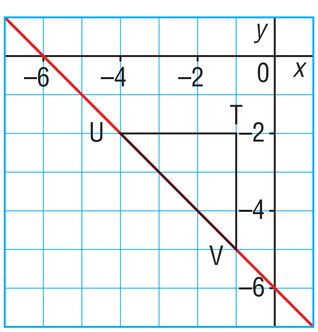
a)



b)



c)



6. State the number of lines of symmetry in each design.

a) a tessellation created by M.C. Escher

b) a Haida button blanket

7. Assessment Focus

a) Draw a triangle on a grid.

b) Choose one side of the triangle as a line of reflection.

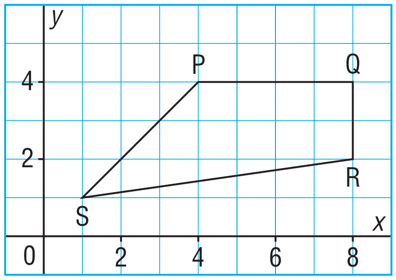
- **i)** Draw the reflection image.
- **ii)** Label the vertices of the shape formed by the original triangle and its image.
- **iii)** Write the coordinates of each vertex.
- **iv)** How many lines of symmetry does the shape have?

c) Repeat part b for each of the other two sides of the triangle. Do you always get the same shape? Explain.

d) Repeat parts a to c for different types of triangles.

e) Which types of triangle always produce a shape that is a quadrilateral with line symmetry? Justify your answer.

8. Quadrilateral PQRS is part of a larger shape.



After each reflection below:

- Draw the image of PQRS.
- Write the coordinates of the larger shape formed by PQRS and its image.
- Describe the larger shape and its symmetry.

a) a reflection in the horizontal line through 4 on the y-axis

b) a reflection in the vertical line through 8 on the x-axis

c) a reflection in the oblique line through (1, 1) and (4, 4)

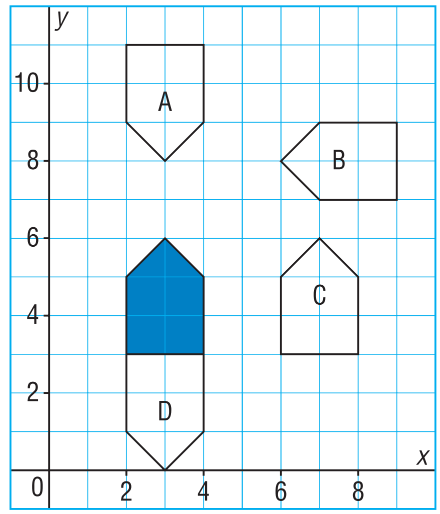
9. a) Graph these points on grid paper:
A(-3, 0), B(-1, 1), C(0, 3),
D(1, 1), E(3, 0).
Join the points to form polygon ABCDE.

b) Reflect the polygon in the x-axis. Draw and label its image.

c) Write the coordinates of the shape formed by the polygon and its image.

d) How many lines of symmetry does this shape have? How do you know?

10. Identify the pentagons that are related to the blue pentagon by a line of reflection. Describe the position of each line of symmetry.



Take It Further

11. a) On a grid, plot the points P(2, 2), Q(6, 2), and R(4, 4). Join the points to form ΔPQR.

b) Reflect ΔPQR in the line through the points (0, 4) and (4, 0). Draw the reflection image.

c) Reflect ΔPQR in the line through the points (0, -4) and (4, 0). Draw the reflection image.

d) Reflect ΔPQR in the x-axis. Draw the reflection image.

e) Look at the shape formed by the triangle and all its images. How many lines of symmetry does this shape have?

Reflect

When you see two shapes on a grid, how can you tell if they are related by a line of reflection?
Include examples of shapes that are related and are not related this way.

GAME

Make Your Own Kaleidoscope

The kaleidoscope was invented in 1816. It uses mirrors placed at different angles to produce patterns with symmetry.

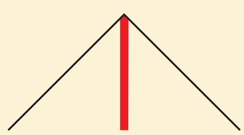
You will need

- 2 small rectangular mirrors
- masking tape

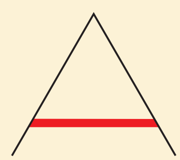
To make a simple kaleidoscope, use masking tape to join two mirrors so they stand at an angle.

Place your mirrors on the arms of each angle below.
Sketch and describe what you see.
Include any lines of symmetry in your sketch.

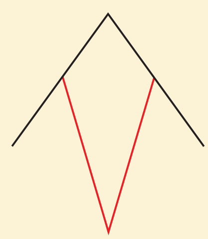
1.



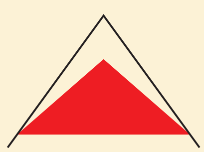
2.



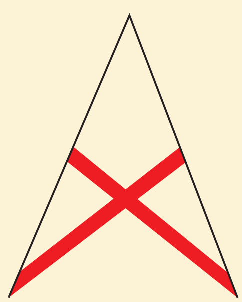
3.



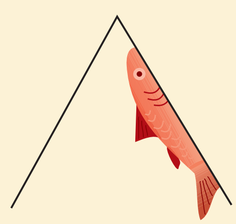
4.



5.



6.



7.6 Rotations and Rotational Symmetry

FOCUS

- Draw and classify shapes with rotational symmetry.

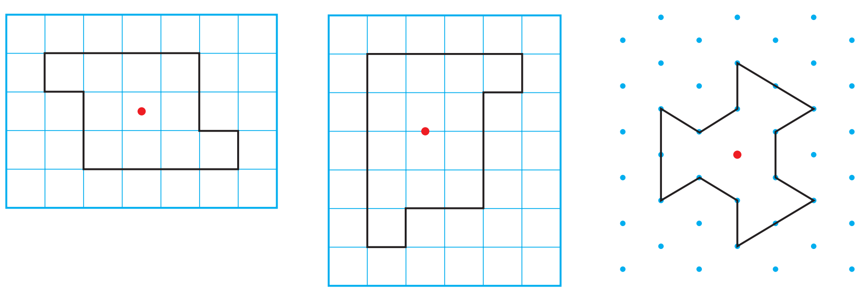
Look at these photographs.
How are the windmills the same?
How are they different?

Investigate

3

You will need a protractor, a sharp pencil, tracing paper, and grid paper or isometric dot paper.

➤ Each of you chooses one of these shapes and copies it on grid paper or dot paper.



➤ Trace your shape and place the tracing to coincide with the shape.
Place a pencil point on the red dot.
Rotate the tracing, counting the number of times the tracing coincides with the original shape, until you make a complete turn.

➤ Repeat the rotation. This time, measure and record the angle you turned the tracing through each time.

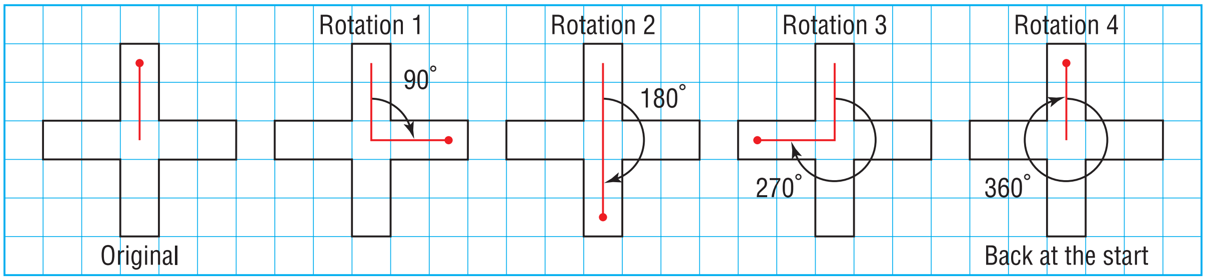
➤ Work together to draw a shape that coincides with itself 4 times as you rotate it.

Reflect & Share

Share your results with another group.
What is the relationship between the number of times the shape coincided with itself and the angle you turned it through each time?

Connect

A tracing of this shape is rotated about its centre. We draw a line segment to help identify the angle the shape turned through before it coincided with itself.



The shape coincided with itself 4 times in one complete turn; that is, during a rotation of 360°.

A shape has **rotational symmetry** when it coincides with itself after a rotation of less than 360° about its centre.
The number of times the shape coincides with itself, during a rotation of 360°, is the **order of rotation.** The shape above has *rotational symmetry of order* 4.

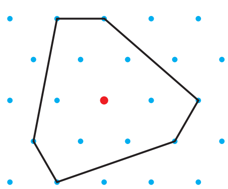
For each match, the shape rotated through 90°.
We say the **angle of rotation symmetry** is 90°. This is $\frac{360°}{4}$.

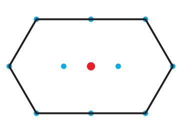
In general, for rotational symmetry:
the angle of rotation symmetry $= \dfrac{360°}{\text{the order of rotation}}$

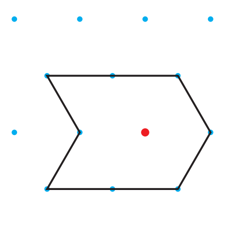
A shape that requires a rotation of 360° to return to its original position does not have rotational symmetry. A shape cannot have rotational symmetry of order 1.

Example 1 Identifying Shapes with Rotational Symmetry

Determine which hexagons below have rotational symmetry.
State the order of rotation and the angle of rotation symmetry.

a) 

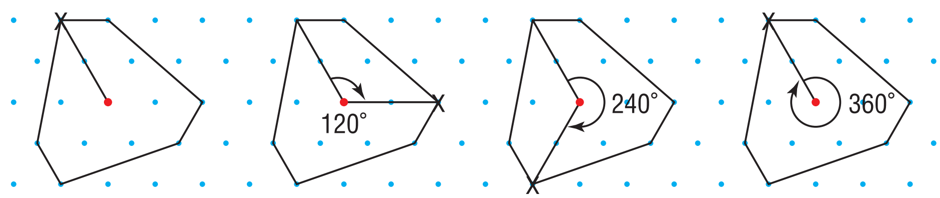
b) 

c) 

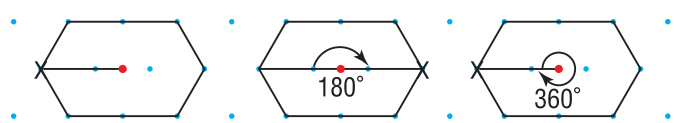
A Solution

For each hexagon:
- Join one vertex to the red dot.
- Trace the hexagon.
- Rotate the tracing about the red dot and record the order of rotation.
- Calculate the angle of rotation symmetry.

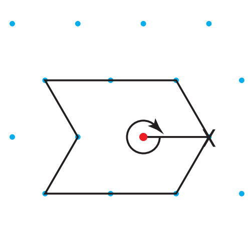
a) The order of rotation is 3.
The angle of rotation symmetry is: $\frac{360°}{3} = 120°$



b) The order of rotation is 2.
The angle of rotation symmetry is: $\frac{360°}{2} = 180°$



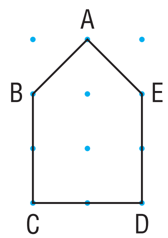
c) This hexagon is rotated one complete turn
before it coincides with itself.
It does not have rotational symmetry.



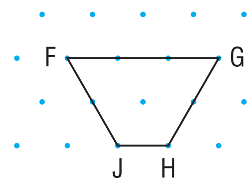
A rotation is another type of transformation.
We use a square grid to draw rotation images after a rotation of 90°, or any multiple of 90°, such as 180° and 270°.
We use isometric dot paper to draw rotation images after a rotation of 60°, or any multiple of 60°, such as 120° and 180°.

Example 2 Drawing Rotation Images

a) Rotate pentagon ABCDE 90° clockwise about vertex E. Draw the rotation image.



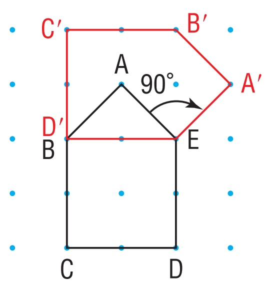
b) Rotate trapezoid FGHJ 120° counterclockwise about vertex F. Draw the rotation image.



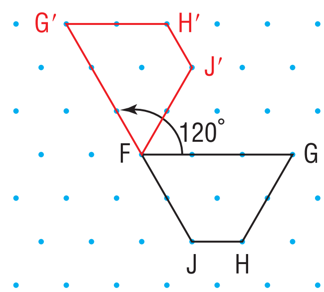
A Solution

Trace each shape and label the vertices on the tracing.

a) Rotate pentagon ABCDE 90° clockwise about E. Side ED moves from being vertical to being horizontal.



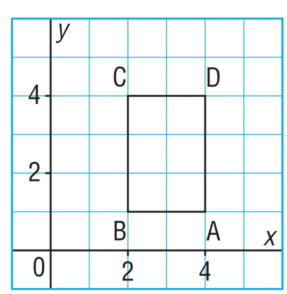
b) Rotate trapezoid FGHJ 120° counterclockwise about F. The angle between FG and FG′ is 120°.



Example 3 Identifying Symmetry after Rotations

a) Rotate rectangle ABCD:
  i) 90° clockwise about vertex A
  ii) 180° clockwise about vertex A
  iii) 270° clockwise about vertex A
  Draw and label each rotation image.

b) Look at the shape formed by the rectangle and all its images. Identify any rotational symmetry in this shape.



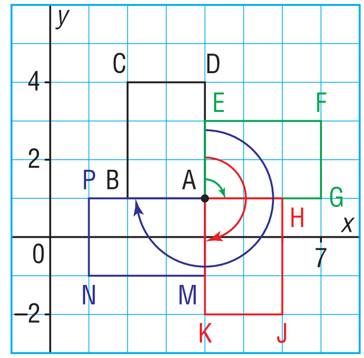
A Solution

a) Trace rectangle ABCD and label the vertices.

i) Rotate ABCD 90° clockwise about A.
Vertical side AD becomes horizontal side AG.
The rotation image is AEFG.

ii) Rotate ABCD 180° clockwise about A.
Vertical side AD becomes vertical side AK.
The rotation image is AHJK.

iii) Rotate ABCD 270° clockwise about A.
Vertical side AD becomes horizontal side AP.
The rotation image is AMNP.



b) The resulting shape BCDEFGHJKMNP has rotational symmetry of order 4 about point A.

Discuss the ideas

1. How do you determine whether a shape has rotational symmetry?

2. How can you determine:

a) the order of rotational symmetry?

b) the angle of rotation symmetry?

3. How is rotational symmetry related to rotation images?

Practice

Check

4. What is the angle of rotation symmetry for a shape with each order of rotational symmetry?

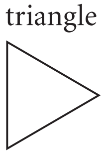
a) 3 **b)** 5 **c)** 9 **d)** 12

5. What is the order of rotational symmetry for each angle of rotation symmetry?

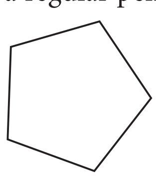
a) 60° **b)** 20° **c)** 45° **d)** 36°

6. What is the order of rotational symmetry and angle of rotation symmetry for each regular polygon?

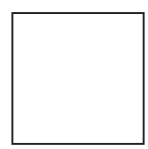
a) an equilateral triangle



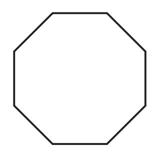
b) a regular pentagon



c) a square



d) a regular octagon

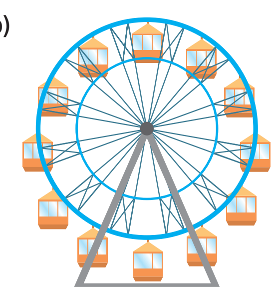


Apply

7. Does each picture have rotational symmetry? If it does, state the order and the angle of rotation symmetry.

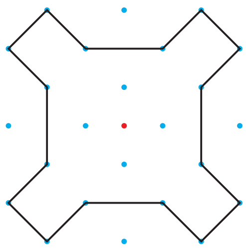
a)

b)

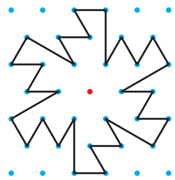


8. Does each shape have rotational symmetry about the red dot? If it does, state the order and the angle of rotation symmetry.

a)

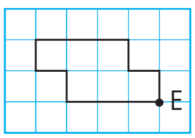


b)

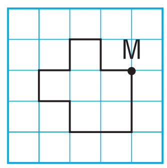


9. Copy each shape on grid paper. Draw the rotation image after each given rotation.

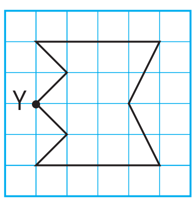
a) 90° clockwise about E



b) 180° about M

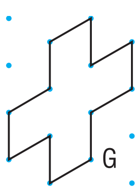


c) 270° counterclockwise about Y

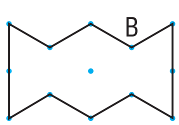


10. Copy each shape on isometric dot paper. Draw the rotation image after each given rotation.

a) 60° clockwise about G



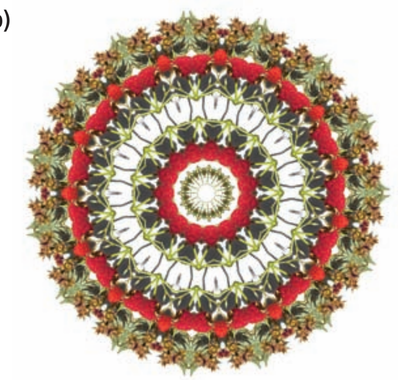
b) 120° counterclockwise about B



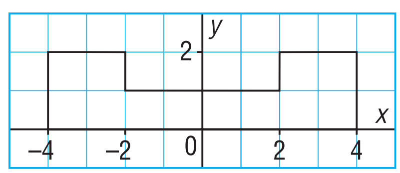
11. Identify and describe any rotational symmetry in each design.

a)

b)



12. This octagon is part of a larger shape that is to be completed by a rotation of 180° about the origin.



a) On a coordinate grid, draw the octagon and its image.

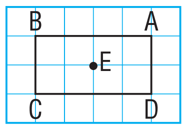
b) Outline the shape formed by the octagon and its image. Describe any rotational symmetry in this shape. Explain why you think the symmetry occurred.

13. **Assessment Focus** Rotate each shape.

a) rectangle ABCD

i) 180° about vertex A

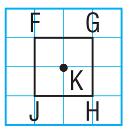
ii) 180° about centre E



b) square FGHJ counterclockwise through

i) 90° about vertex F

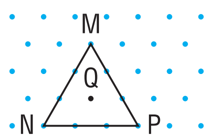
ii) 90° about centre K



c) equilateral triangle MNP clockwise through

i) 120° about vertex M

ii) 120° about centre Q

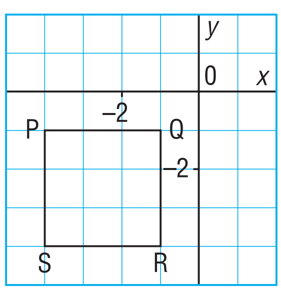


d) How are the images in each of parts a, b, and c the same? How are they different? Explain what you notice.

14. a) Rotate square PQRS clockwise about vertex P through:

i) 90° ii) 180° iii) 270°

Draw and label each rotation image.

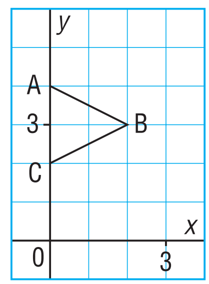


b) Outline the shape formed by the square and all its images. Identify any rotational symmetry. Explain what you notice.

15. Triangle ABC is part of a larger shape that is to be completed by three rotations.

a) Rotate ΔABC clockwise about vertex C through: i) 90° ii) 180° iii) 270°

Draw and label each rotation image.



b) List the coordinates of the vertices of the larger shape formed by the triangle and its images. Describe any rotational symmetry.

Take It Further

16. a) Draw a polygon on a coordinate grid. Choose an angle of rotation and a centre of rotation to complete a larger polygon with order of rotation: i) 2 ii) 4

List the coordinates of the centre of rotation, and the vertices of the larger polygon.

b) Draw a polygon on isometric dot paper. Choose an angle of rotation and a centre of rotation to complete a larger polygon with order of rotation: i) 3 ii) 6

Reflect

How do you decide if a given shape has rotational symmetry?
If it does, how do you determine the order of rotation and the angle of rotation symmetry?
Include an example in your explanation.

7.7 Identifying Types of Symmetry on the Cartesian Plane

FOCUS

- Identify and classify line and rotational symmetry.

What symmetry do you see in each picture?

Investigate

3

You will need grid paper and tracing paper.

➤ Plot these points on a coordinate grid: A(1, 3), B(3, 1), and C(5, 5)
Join the points to form ΔABC.

➤ Each of you chooses one of these transformations:
- a translation 2 units right and 2 units down
- a rotation of 180° about vertex C
- a reflection in a line through AB

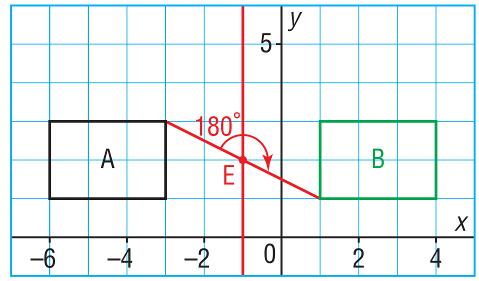
Draw the image for the transformation you chose.
Record the coordinates of each vertex on the image.
On a separate piece of paper, record any symmetry in the triangle and its image.

➤ Trade grids with a member of your group.
Identify any symmetry in the triangle and its image.

Reflect & Share

Compare the types of symmetry you found.
Did any grid show both rotational symmetry and line symmetry?
Explain why both types of symmetry occurred.
Which grid showed only one type of symmetry?

Connect



On this grid, rectangle A has been rotated 180° about E(−1, 2) to produce its image, rectangle B.
We can extend our meaning of line symmetry to relate the two rectangles.
The line through −1 on the x-axis is a line of symmetry for the two rectangles.
Each point on rectangle A has a corresponding point on rectangle B.
These points are equidistant from the line of symmetry.

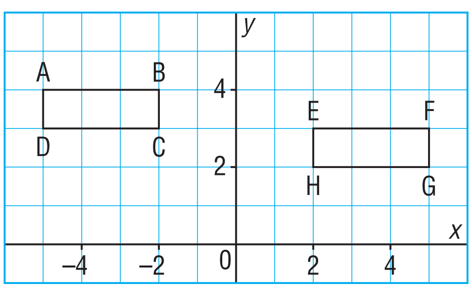
When a shape and its transformation image are drawn, the resulting diagram may show:

- no symmetry
- line symmetry
- rotational symmetry
- both line symmetry and rotational symmetry

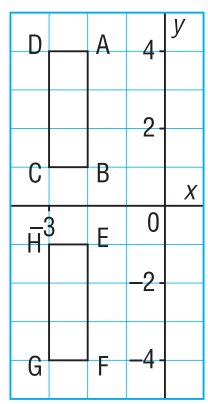
Example 1 Determining whether Shapes Are Related by Symmetry

For each pair of rectangles ABCD and EFGH, determine whether they are related by symmetry.

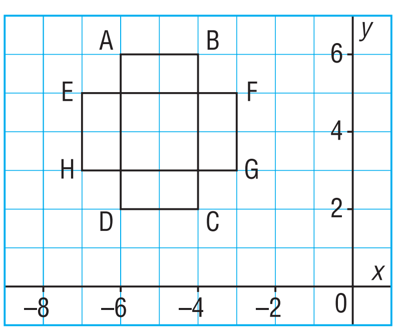
a)



b)

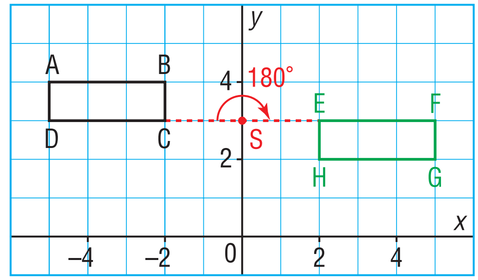


c)

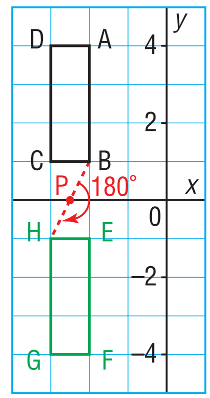


A Solution

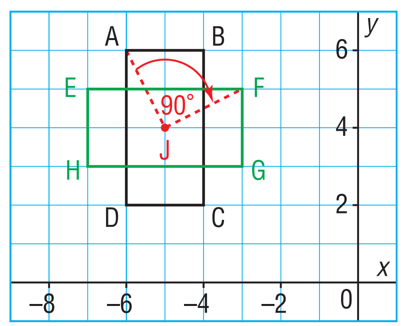
a) There is no line on which a mirror can be placed so that one rectangle is the reflection image of the other. So, the rectangles are not related by line symmetry.
Trace the rectangles. Use guess and check to determine if a centre of rotation exists.
When ABCD is rotated 180° about the point S(0, 3), ABCD coincides with GHEF.
So, the rectangles are related by rotational symmetry of order 2 about S(0, 3).



b) Each point on ABCD has a corresponding point on EFGH. These points are equidistant from the x-axis.
So, the two rectangles are related by line symmetry; the x-axis is the line of symmetry.
Trace the rectangles. Use guess and check to determine if a centre of rotation exists.
When a tracing of ABCD is rotated 180° about the point P(−2.5, 0), ABCD coincides with GHEF.
So, the two rectangles are related by rotational symmetry.



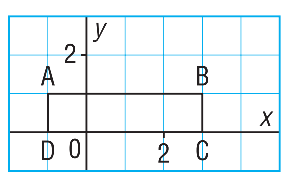
c) When ABCD is rotated 90° clockwise about point J(−5, 4), ABCD coincides with FGHE.
Then, the polygon formed by both rectangles together has rotational symmetry of order 4 about point J. So, the two rectangles are related by rotational symmetry.



Example 2 Identifying Symmetry in a Shape and Its Transformation Image

Draw the image of rectangle ABCD after each transformation.
Write the coordinates of each vertex and its image.
Identify and describe the type of symmetry that results.

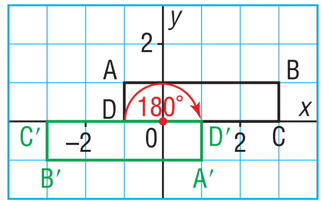
a) a rotation of 180° about the origin
b) a reflection in the x-axis
c) a translation 4 units right and 1 unit down



Solution

a) Use tracing paper to rotate ABCD 180° about the origin.

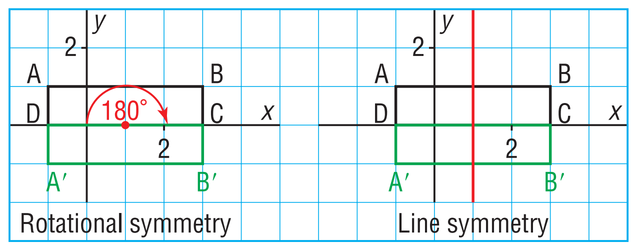
Point	Image
A(−1, 1)	A′(1, −1)
B(3, 1)	B′(−3, −1)
C(3, 0)	C′(−3, 0)
D(−1, 0)	D′(1, 0)



The octagon ABCD′A′B′C′D, formed by both rectangles together, has rotational symmetry of order 2 about the origin, and no line symmetry.

b) Reflect ABCD in the x-axis.

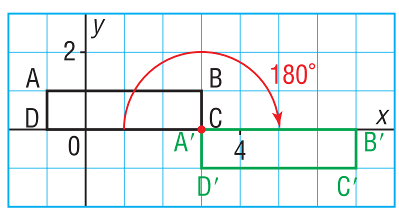
Point	Image
A(−1, 1)	A′(−1, −1)
B(3, 1)	B′(3, −1)
C(3, 0)	C(3, 0)
D(−1, 0)	D(−1, 0)



The rectangle ABB′A′, formed by both rectangles, has rotational symmetry of order 2 about the point (1, 0). It also has 2 lines of symmetry: the x-axis and the vertical line through 1 on the x-axis.

c) Translate ABCD 4 units right and 1 unit down.

Point	Image
A(−1, 1)	A′(3, 0)
B(3, 1)	B′(7, 0)
C(3, 0)	C′(7, −1)
D(−1, 0)	D′(3, −1)



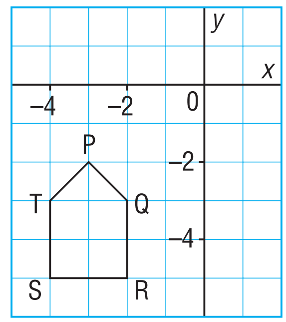
The two rectangles do not form a shape; but they have a common vertex at C (or A′). The two rectangles are related by rotational symmetry of order 2 about the point C(3, 0). There is no line of symmetry relating the rectangles.

In *Example 2*, we could write the translation 4 units right and 1 unit down in a shorter form as R4, D1. In this shorter form, a translation of 7 units left and 2 units up would be written as L7, U2.

Example 3 Identifying Symmetry in Shapes and their Translation Images

Draw the image of pentagon PQRST after each translation below.
Label the vertices of the pentagon and its image, and list their coordinates.
If each diagram has symmetry, describe it.
If each diagram does not have symmetry, explain how you know.

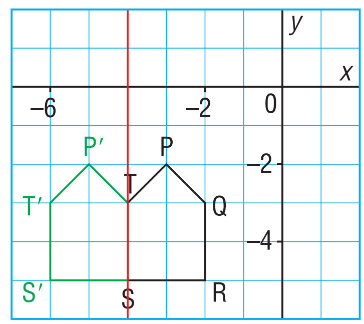
a) a translation L2 **b)** a translation L2, D3



A Solution

a) Translate each vertex of pentagon PQRST 2 units left.

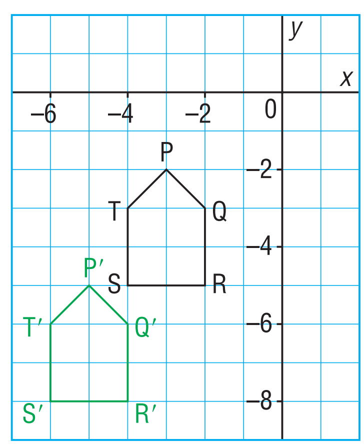
Point	Image
P(−3, −2)	P′(−5, −2)
Q(−2, −3)	T(−4, −3)
R(−2, −5)	S(−4, −5)
S(−4, −5)	S′(−6, −5)
T(−4, −3)	T′(−6, −3)



The diagram has line symmetry because the vertical line through ST is a line of reflection.
The diagram does not have rotational symmetry because there is no point about which it can be rotated so that it coincides with itself.

b) Translate each vertex of pentagon PQRST 2 units left and 3 units down.

Point	Image
P(−3, −2)	P′(−5, −5)
Q(−2, −3)	Q′(−4, −6)
R(−2, −5)	R′(−4, −8)
S(−4, −5)	S′(−6, −8)
T(−4, −3)	T′(−6, −6)



The diagram does not have line symmetry because there is no line on which a mirror can be placed so that one pentagon is the reflection image of the other.
The diagram does not have rotational symmetry because there is no point about which it can be rotated so that it coincides with itself.

Discuss the ideas

1. How can you tell if two shapes are related by line symmetry?

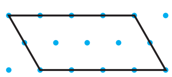
2. How can you tell if two shapes are related by rotational symmetry?

Practice

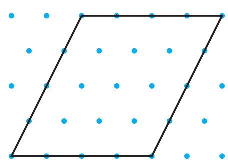
Check

3. Describe the rotational symmetry and line symmetry of each shape.

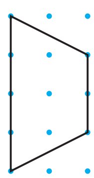
a) a parallelogram



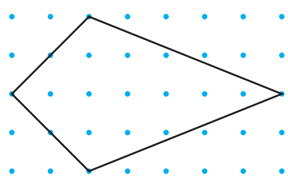
b) a rhombus



c) an isosceles trapezoid



d) a kite



4. Describe the rotational symmetry and line symmetry of each wheel cover. On a copy of the wheel covers, mark the centre of rotation and the line of reflection.

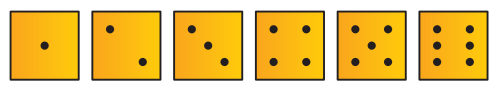
a)

b)

c)

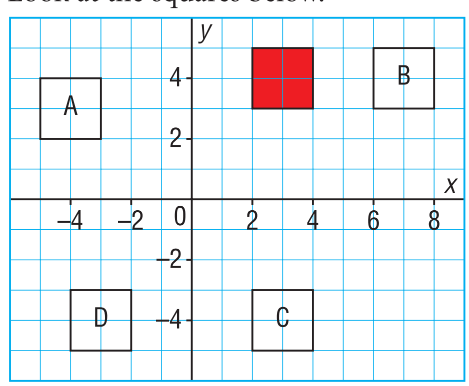
d)

5. Describe the symmetry of each face of a die. Copy each face. Mark the centre of rotation and the lines of symmetry.



Apply

6. Look at the squares below.



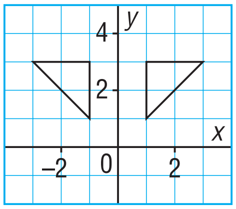
Which of squares A, B, C, and D are related to the red square:

a) by rotational symmetry about the origin?

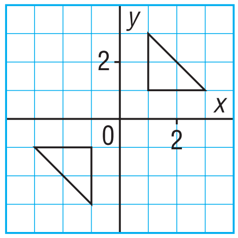
b) by line symmetry?

7. For each diagram, determine whether the two polygons are related by line symmetry, by rotational symmetry about the origin, or by both.

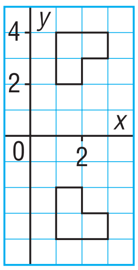
a)



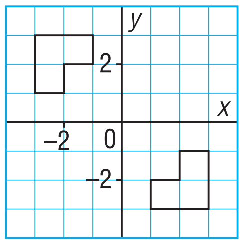
b)



c)

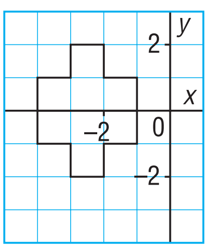


d)

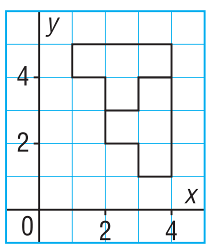


8. For each diagram, determine whether the two octagons are related by line symmetry, by rotational symmetry, by both types of symmetry, or by neither.

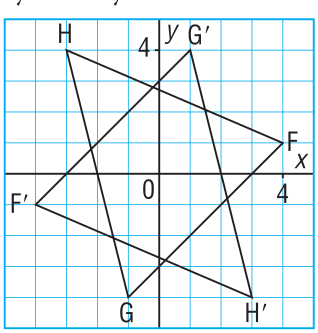
a)



b)



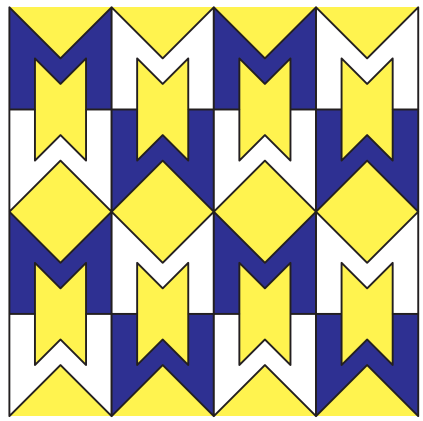
9. Triangle F′G′H′ is the image of ΔFGH after a rotation about the origin. Identify any symmetry.



10. Identify and describe the types of symmetry in each piece of artwork.

a)

b)



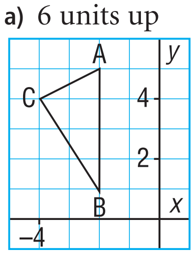
11. Copy each shape on grid paper.

- Draw the image after the translation given.
- Label each vertex with its coordinates.
- Does each diagram have line and rotational symmetry?

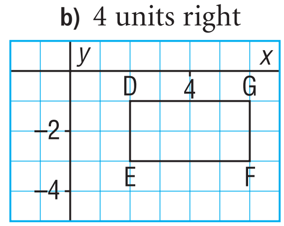
  If your answer is yes, describe the symmetry.

  If your answer is no, describe how you know.

a) 6 units up



b) 4 units right



12. Assessment Focus

a) On a grid, draw ΔCDE with vertices $C(2, 3)$, $D(-2, -1)$, and $E(3, -2)$.

b) Draw the translation image $\Delta C'D'E'$ after the translation R1, U3.

c) Label all the vertices with their ordered pairs.

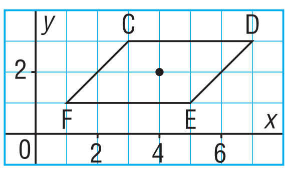
d) Explain why the translation does not result in line or rotational symmetry.

e) Find a translation that does result in one type of symmetry. Draw the image. How do you know the diagram has symmetry?

Show your work.

13. a) Draw the image of parallelogram CDEF after each transformation below.

b) The parallelogram and its image form a diagram. If each diagram has symmetry, describe it. If each diagram does not have symmetry, describe how you know.



i) a rotation of 90° clockwise about (4, 2)

ii) a reflection in the horizontal line through 1 on the y-axis

iii) a translation R4

14. The digits 0 to 9 on a digital clock are made up from horizontal and vertical segments.

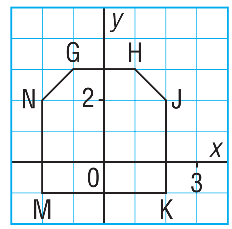
a) Sketch each digit on dot paper. Identify any symmetry it has.

b) For each digit with line symmetry, plot a part of the digit on grid paper and draw a line of symmetry so that the digit can be completed by a reflection.

c) For each digit with rotational symmetry, plot a part of the digit on grid paper. Locate the point about which the digit can be completed by a rotation.

d) Is there a pair of digits that are related by line or rotational symmetry? Justify your answer by plotting the digits on a Cartesian plane.

15. This hexagon is part of a larger shape that is completed by rotating the hexagon 180° about the origin.



a) Draw the rotation image.

b) List the coordinates of the vertices of the larger shape.

c) Describe the symmetry in the larger shape.

Take It Further

16. The 24-hour clock represents midnight as 00:00 and three-thirty A.M. as 03:30. The time 03:30 has line symmetry with a horizontal line of reflection. List as many times from midnight onward that have line symmetry, rotational symmetry, or both. Describe the symmetry for each time you find.

Reflect

When you see a shape and its transformation image on a grid, how do you identify line symmetry and rotational symmetry? Include examples in your explanation.

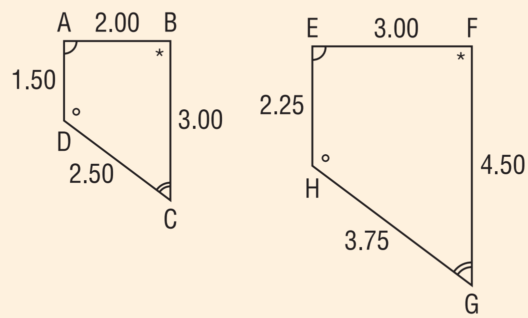
Study Guide

Scale Diagrams

For an enlargement or reduction, the scale factor is: $\dfrac{\text{Length on scale diagram}}{\text{Length on original diagram}}$

An enlargement has a scale factor > 1. A reduction has a scale factor < 1.

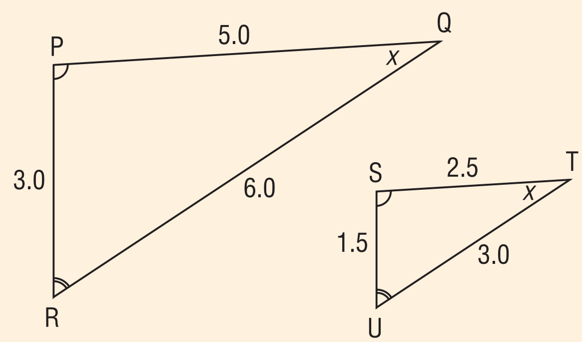
Similar Polygons



Similar polygons are related by an enlargement or a reduction. When two polygons are similar:

- their corresponding angles are equal:
  $\angle A = \angle E$; $\angle B = \angle F$; $\angle C = \angle G$; $\angle D = \angle H$
  and
- their corresponding sides are proportional:
  $\dfrac{AB}{EF} = \dfrac{BC}{FG} = \dfrac{CD}{GH} = \dfrac{DA}{HE}$
  Any of the ratios $\dfrac{AB}{EF}$, $\dfrac{BC}{FG}$, $\dfrac{CD}{GH}$, and $\dfrac{DA}{HE}$ is the scale factor.

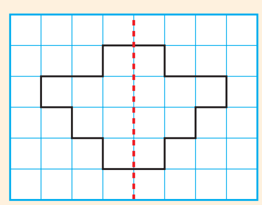
Similar Triangles



When we check whether two triangles are similar:

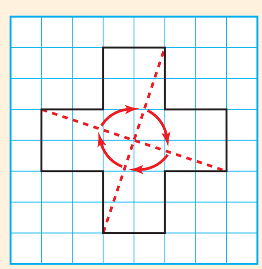
- their corresponding angles must be equal:
  $\angle P = \angle S$ and $\angle Q = \angle T$ and $\angle R = \angle U$
  or
- their corresponding sides must be proportional:
  $\dfrac{PQ}{ST} = \dfrac{QR}{TU} = \dfrac{PR}{SU}$
  Any of the ratios $\dfrac{PQ}{ST}$, $\dfrac{QR}{TU}$, and $\dfrac{PR}{SU}$ is the scale factor.

Line Symmetry



A shape has line symmetry when a line divides the shape into two congruent parts so that one part is the image of the other part after a reflection in the line of symmetry.

Rotational Symmetry



A shape has rotational symmetry when it coincides with itself after a rotation of less than 360° about its centre. The number of times the shape coincides with itself is the order of rotation.

The angle of rotation symmetry $= \dfrac{360°}{\text{the order of rotation}}$

Review

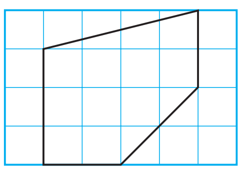
7.1

1. This photo of participants in the Arctic Winter Games is to be enlarged.

Measure the photo. What are the dimensions of the enlargement for each scale factor?

a) 3 **b)** 2.5 **c)** $\frac{3}{2}$ **d)** $\frac{21}{5}$

2. Draw this pentagon on 1-cm grid paper. Then draw an enlargement of the shape with a scale factor of 2.5.



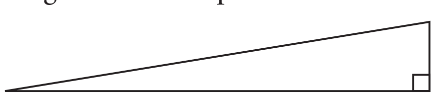
7.2

3. A full-size pool table has dimensions approximately 270 cm by 138 cm. A model of a pool table has dimensions 180 cm by 92 cm.

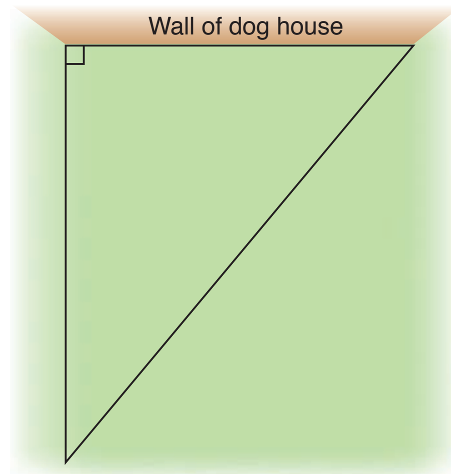
a) What is the scale factor for this reduction?

b) A standard-size pool cue is about 144 cm long. What is the length of a model of this pool cue with the scale factor from part a?

4. Here is a scale diagram of a ramp. The height of the ramp is 1.8 m. Measure the lengths on the scale diagram. What is the length of the ramp?

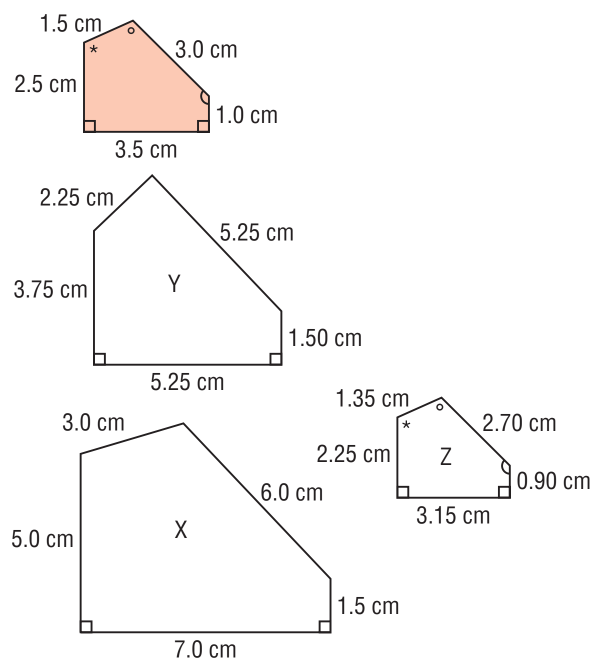


5. Gina plans to build a triangular dog run against one side of a dog house. Here is a scale diagram of the run. The wall of the dog house is 2 m long. Calculate the lengths of the other two sides of the dog run.

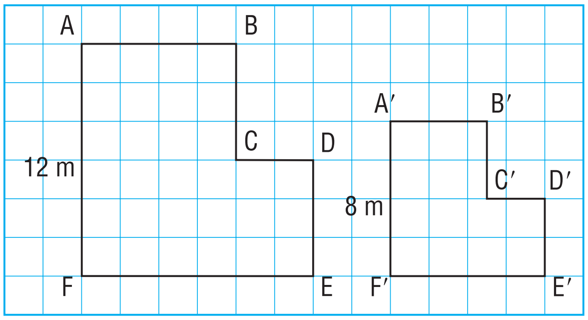


7.3

6. Which pentagon is similar to the red pentagon? Justify your answer.



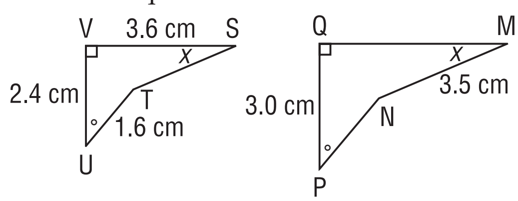
7. These two courtyards are similar.



Determine each length.

a) BC **b)** B′C′ **c)** A′B′

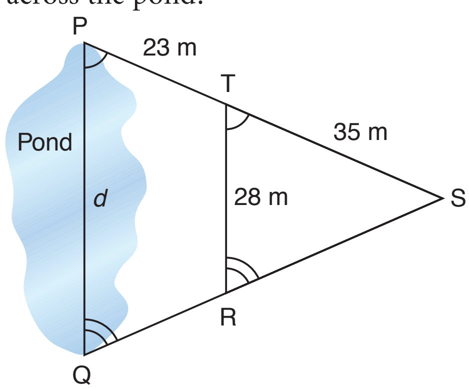
8. These two quadrilaterals are similar.



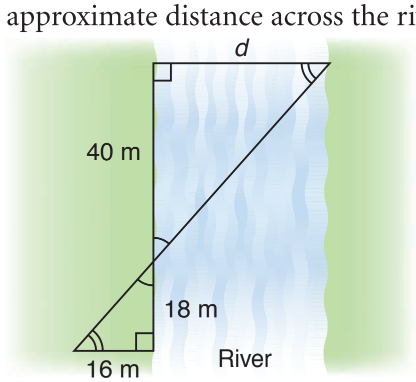
Calculate the length of: **a)** PN **b)** TS

7.4

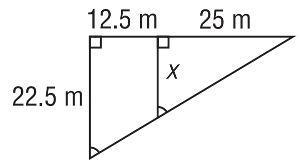
9. To determine the distance, *d*, across a pond, Ari uses this diagram. What is the distance across the pond?



10. This scale diagram shows a surveyor's measurements taken to determine the distance across a river. What is the approximate distance across the river?



11. How can you use similar triangles to calculate the distance x in this scale diagram?



7.5

12. Which of these traffic signs have line symmetry? How many lines of symmetry in each case?

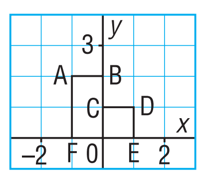
a)


b)


c)


d)


13. Hexagon ABCDEF is a part of a larger shape. Copy the hexagon on a grid.



a) Complete the shape by reflecting the hexagon:

i) in the y-axis

ii) in the x-axis

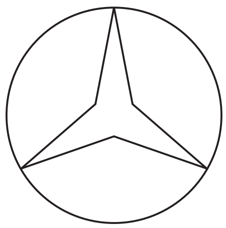
iii) in the line through $(-2, -1)$ and $(2, 3)$

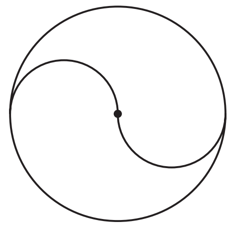
b) Complete the shape with a translation R2.

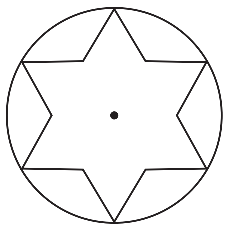
c) List the ordered pairs of the vertices of each completed shape.

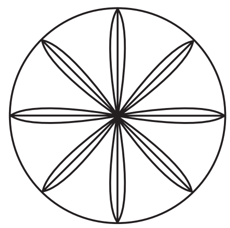
d) State whether each completed shape has line symmetry.

7.6 **14.** What is the order of rotational symmetry of each shape? How do you know?

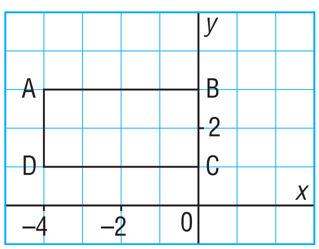
a)


b)


c)


d)


15. Rectangle ABCD is part of a larger shape that is to be completed by a transformation image.



a) Rotate rectangle ABCD as indicated, then draw and label each image.

i) 90° counterclockwise about the point $(-4, 2)$

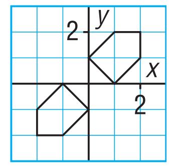
ii) 180° about vertex B

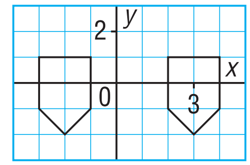
iii) 270° counterclockwise about the point $(-2, 2)$

b) Which diagrams in part a have rotational symmetry? How do you know?

7.7 **16.** Look at the diagrams in question 15. Which diagrams have line symmetry? How do you know?

17. For each diagram, determine whether the two pentagons are related by any symmetry. Describe each type of symmetry.

a)


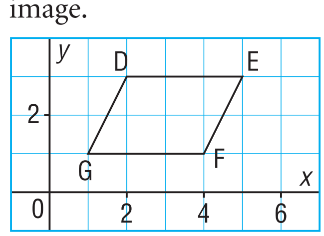
b)


18. Identify and describe the types of symmetry in each piece of artwork.

a)


b)


19. a) Translate quadrilateral DEFG as indicated, then draw and label each image.

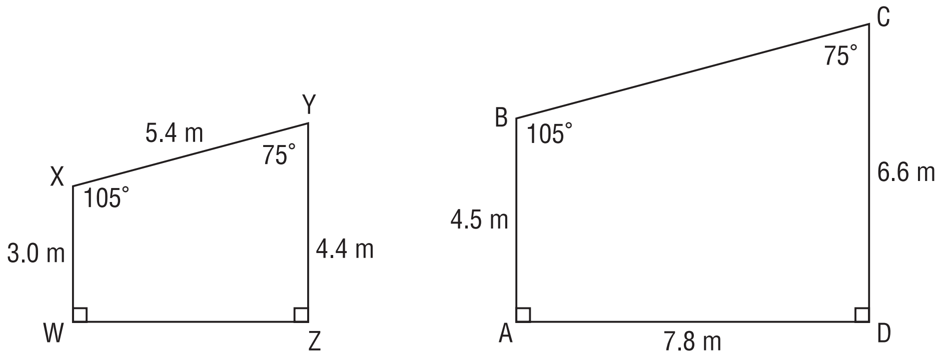


i) L4, D2 ii) R1, U2

b) Does each translation result in line symmetry or rotational symmetry? If your answer is yes, describe the symmetry. If your answer is no, explain why there is no symmetry.

Practice Test

1. These two quadrilaterals are similar.



a) Calculate the length of BC.
b) Calculate the length of WZ.
c) Draw an enlargement of quadrilateral WXYZ with scale factor 2.
d) Draw a reduction of quadrilateral ABCD with scale factor $\frac{1}{3}$.

2. Scott wants to calculate the height of a tree. His friend measures Scott's shadow as 3.15 m. At the same time, the shadow of the tree is 6.30 m. Scott knows that he is 1.7 m tall.
a) Sketch two triangles Scott could use to calculate the height of the tree.
b) How do you know the triangles are similar?
c) What is the height of the tree?

3. Use isometric dot paper or grid paper.
a) Draw these shapes: equilateral triangle, square, rectangle, parallelogram, trapezoid, kite, and regular hexagon
b) For each shape in part a:
  i) Draw its lines of symmetry.
  ii) State the order and angle of rotation symmetry.
c) Draw a shape that has line symmetry but not rotational symmetry.
d) Draw a shape that has rotational symmetry but not line symmetry.

4. Plot these points on a grid: A(2, 1), B(1, 2), C(1, 4), D(2, 5), E(3, 4), F(3, 2)
For each transformation below:
  i) Draw the transformation image.
  ii) Record the coordinates of its vertices.
  iii) Describe the symmetry of the diagram formed by the original shape and its image.
a) a rotation of 90° clockwise about the point G(2, 3)
b) a translation R2
c) a reflection in the line $y = 2$

Unit Problem — Designing a Flag

Part 1

At sea, flags are used to display messages or warnings.
Here are some nautical flags.

➤ Describe the symmetries of each flag in as much detail as possible.

➤ Classify the flags according to the numbers of lines of symmetry.

Part 2

Design your own flag.
The flag may be for a country, an organization, or it may be a flag with a message. It must have line symmetry and rotational symmetry.

➤ Describe the symmetries in your flag.

The actual flag must be at least 3 m by 2 m.

➤ Draw a scale diagram of your flag, including the scale factor you used.

➤ Describe what your flag will be used for.

Your work should show:

- a description and classification of the symmetries of the nautical flags
- a scale diagram of your flag, in colour, including the scale factor
- a description of the symmetries in your flag
- a description of what your flag will be used for

Reflect on Your Learning

How does knowledge of enlargements and reductions in scale diagrams help you understand similar polygons?
How are line symmetry and rotational symmetry related to transformations on a grid?