Surface Areas of Objects Made from Right Rectangular Prisms

FOCUS

• Determine the surface areas of composite objects made from cubes and other right rectangular prisms. These cube houses were built in Rotterdam, Netherlands. Suppose you wanted to determine the surface area of one of these houses. What would you need to know?



Investigate

Each of you needs 5 linking cubes.

Assume each face of a linking cube has area 1 unit².

What is the surface area of 1 cube?
 Put 2 cubes together to make a "train."
 What is the surface area of the train?
 Place another cube at one end of your train.
 What is its surface area now?

Continue to place cubes at one end of the train,

and determine its surface area.

- Copy and complete this table.
- What patterns do you see in the table?

What happens to the surface area each time you place another cube on the train? Explain why the surface area changes this way.

With the 5 cubes, build an object that is different from the train and different from your partner's object.

Determine its surface area.

Compare the surface area of your object with that of your partner's object.

Number of Cubes	Surface Area (square units)
1	
2	
3	
4	
5	

2



Compare your objects with those of another pair of students who made different objects. Are any of the surface areas different? If your answer is yes, explain how they can be different when all the objects are made with 5 cubes.

Connect

Here is an object made from 4 unit cubes. Each face of a cube is a square with area 1 unit².



Here are 2 strategies for determining the surface area of the object.

 Count the square faces of all the cubes, then subtract 2 faces for each surface where the cubes are joined. We say the faces *overlap*.

The object has 4 cubes. Each cube has 6 faces. So, the number of faces is: $6 \times 4 = 24$ There are 3 places where the faces overlap, so subtract: 3×2 , or 6 faces The surface area, in square units, is: 24 - 6 = 18





The surface area, in square units, is:

4 + 4 + 3 + 3 + 2 + 2 = 18



An object like that on page 26 is called a **composite object** because it is made up, or *composed*, of other objects.

Example 1 Determining the Surface Area of a Composite Object Made from Cubes

Determine the surface area of this composite object. Each cube has edge length 2 cm.



Solutions

Method 1

Count the squares on each of the 6 views:



Each of the front, back, top, and bottom views has 4 squares.

Each of the right and left views has 3 squares. The surface area, in squares, is: $(4 \times 4) + (3 \times 2) = 22$

Each square has area: $2 \text{ cm} \times 2 \text{ cm} = 4 \text{ cm}^2$

So, the surface area is: $22 \times 4 \text{ cm}^2 = 88 \text{ cm}^2$

Method 2

The composite object has 5 cubes. Each cube has 6 square faces. So, the total number of squares is: $5 \times 6 = 30$



The cubes overlap at 4 places, so there are 4×2 , or 8 squares that are not part of the surface area. The surface area, in squares, is: 30 - 8 = 22Each square has area: $2 \text{ cm} \times 2 \text{ cm} = 4 \text{ cm}^2$ So, the surface area is: $22 \times 4 \text{ cm}^2 = 88 \text{ cm}^2$

We can use the surface area of composite objects to solve problems outside the classroom.

Example 2

Determining the Surface Area of a Composite Object Made from Right Rectangular Prisms

Renee uses 3 pieces of foam to make this chair. Each piece of foam is a right rectangular prism with dimensions 60 cm by 20 cm by 20 cm. Can Renee cover the chair with 2 m² of fabric? Explain.



A Solution

Convert each measurement to metres, then the surface area is measured in square metres. 60 cm = 0.6 m 20 cm = 0.2 m

Determine the surface area of the rectangular prism that is the base of the chair.

Area of top and bottom faces: $2(0.6 \times 0.4) = 0.48$ Area of front and back faces: $2(0.6 \times 0.2) = 0.24$ Area of left and right faces: $2(0.2 \times 0.4) = 0.16$ Surface area of the base of the chair: 0.48 + 0.24 + 0.16 = 0.88

Determine the surface area of the rectangular prism that is the back rest.

Area of top, bottom, front, and back: $4(0.6 \times 0.2) = 0.48$ Area of left and right faces: $2(0.2 \times 0.2) = 0.08$ Surface area of back rest: 0.48 + 0.08 = 0.56





Add the two surface areas, then subtract twice the area of the overlap because neither of these areas is part of the surface area of the chair: $0.88 + 0.56 - 2(0.6 \times 0.2) = 1.44 - 0.24$ = 1.2

The surface area that is to be covered in fabric is 1.2 m². Since 2 m² > 1.2 m², Renee can cover the chair with 2 m² of fabric.

Example 3 Solving Problems Involving the Surface Area of a Composite Object

A warehouse measures 60 m by 30 m by 20 m. An office attached to one wall of the warehouse measures 20 m by 20 m by 10 m.

- a) Determine the surface area of the building.
- b) A contractor quotes to paint the exterior of the building at a rate of \$2.50/m².

These parts of the building are not to be painted: the 2 roofs; the office door with area 2 m²; 3 loading doors, each measuring 10 m by 15 m; and 4 windows on the office, each with area 1 m². How much would it cost to paint the building?



A Solution

The surface area is measured in square metres.

a) The 4 walls and roof of the warehouse form its surface area. Area of roof: 60 × 30 = 1800 Area of left and right side walls: 2(30 × 20) = 1200 Area of the front and back walls: 2(60 × 20) = 2400 So, the surface area of the warehouse is: 1800 + 1200 + 2400 = 5400

The 3 walls and roof of the office form its surface area. Area of roof: $20 \times 20 = 400$ Area of front, left, and right side walls: $3(20 \times 10) = 600$ So, the surface area of the office is: 400 + 600 = 1000

For the surface area of the building, add the surface areas of the warehouse and the office, then subtract the area of the overlap. The area of the overlap, which is the back of the office, is: $20 \times 10 = 200$ So, the surface area of the building is: $5400 \text{ m}^2 + 1000 \text{ m}^2 - 200 \text{ m}^2 = 6200 \text{ m}^2$

b) To calculate the area to be painted, subtract the areas of the roofs, doors, and windows from the surface area of the building. Area of roofs: 1800 + 400 = 2200
Area of loading doors: 3(10 × 15) = 450
Area of office door and windows: 2 + 4(1) = 6
So, the area to be painted is: 6200 m² - 2200 m² - 450 m² - 6 m² = 3544 m²
The cost to paint the building is: 3544 × \$2.50 = \$8860.00



- **1.** When a composite object is made from right rectangular prisms, why is the surface area of the object not the sum of the surface areas of the individual prisms?
- **2.** The surface area of an object is the area of a net of the object. How would drawing a net help you determine the surface area of a composite object?
- **3.** In *Example 3*, why are the bases of the warehouse and office not included in the surface area?

Practice

Check

 Make each composite object with cubes. Assume each face of a cube has area 1 unit². Determine the surface area of each composite object.







Apply

5. These are 1-cm cubes.



- a) Determine the surface area of the composite object formed by placing cube 4 on top of each indicated cube.
 i) cube 1 ii) cube 2 iii) cube 3
 b) Why are the surface areas in part a equal?
- **6.** These are 1-cm cubes.



- a) Determine the surface area of the composite object formed by placing cube 5 on top of each indicated cube.
 i) cube 1 ii) cube 2 iii) cube 3
- **b)** Why are all the surface areas in part a not equal?

7. Why could you not use 6 views to determine the surface area of this composite object?



8. Determine the surface area of each composite object.What effect does the overlap have on the



- **9.** Work with a partner. Tape a tissue box on a shoebox to form a composite object.
 - a) What is the area of the overlap? How did you calculate it?

1.5 cm

b) Determine the surface area of the object. How did you use the area of the overlap in your calculation? **10. Assessment Focus** A garage has the dimensions shown. The attached shed has the same height as the garage, but is one-half as long and one-half as wide.



- a) What is the surface area of the building?
- b) Vinyl siding costs \$15/m². The doors, windows, and roof will not be covered with siding. How much will it cost to cover this building with siding?
- **11.** This is a floor plan of a building that is 8 m tall. It has a flat roof. What is the surface area of the building, including its roof?



- **12.** Use 27 small cubes to build a large cube.
 - a) Determine and record its surface area.
 - b) How many ways can you remove one cube without changing the surface area? Explain your work.
 - c) Suppose you painted the large cube. How many small cubes would have paint on:
 i) exactly 1 face? ii) exactly 2 faces?
 iii) exactly 3 faces? iv) 0 faces?
 v) more than 3 faces?
 - How could you check your answers?

13. Every January, the Ice Magic Festival is held at Chateau Lake Louise in Banff National Park. An ice castle is constructed from huge blocks of ice.



- a) Suppose you have 30 blocks of ice measuring 25 cm by 50 cm by 100 cm. Sketch a castle with no roof that could be built with some or all of these blocks.
- **b)** Determine the surface area of your castle, inside and out.

Take It Further

14. Use 6 centimetre cubes.

- a) Build a composite object. Sketch the object, then determine and record its surface area.
- b) Use the cubes to build other objects with different surface areas. Sketch each object and record its surface area.
- **c)** Determine all the different surface areas for a composite object of 6 cubes.
- d) Describe the object with the greatest surface area. Describe the object with the least surface area.

- Use centimetre cubes. Build, then sketch all possible composite objects that have a surface area of 16 cm².
- 16. A pyramid-like structure is made with 1-m³ wooden cubes. The bottom layer of the structure is a rectangular prism with a square base and a volume of 25 m³. The next layer has a volume of 16 m³. The pattern of layers continues until the top layer, which has a volume of 1 m³. Determine the surface area of the structure. Describe any patterns you find.
- 17. The SOMA Puzzle was invented by a Danish poet and scientist named Piet Hein in 1936. The object of the puzzle is to arrange these 7 pieces to form one large cube:



- a) Determine the surface area of each piece.
- b) Use linking cubes to make your own pieces and arrange them to form a large cube.
- c) Suppose you painted the large cube. How many faces of the original 7 pieces would not be painted? How do you know?

Reflect

Why is it important to consider the areas of overlap when determining the surface area of a composite object? Include an example in your explanation.

Surface Areas of Other Composite Objects

A student designed this stand for a table lamp. How could the student determine the surface area of this stand? What would he need to know?

FOCUS

• Determine the surface areas of composite objects made from right prisms and right cylinders.

Investigate

To meet safety regulations, a wheelchair ramp must be followed by a landing. This wheelchair ramp and landing lead into an office building. Calculate the surface area of the ramp and landing.



We use the strategies from Lesson 1.3 to determine the surface area of a composite object made from right cylinders and right triangular prisms. That is, consider each prism or cylinder separately, add their surface areas, then account for the overlap.

2

For composite objects involving right prisms, we can use word formulas to determine the surface areas of the prisms.

- ► A right rectangular prism has 3 pairs of congruent faces:
 - the top and bottom faces
 - the front and back faces
 - the left side and right side faces



The surface area is the sum of the areas of the faces: Surface area = $2 \times$ area of top face + $2 \times$ area of front face + $2 \times$ area of side face

- ► A right triangular prism has 5 faces:
 - 2 congruent triangular bases
 - 3 rectangular faces

The surface area is the sum of the areas of the triangular bases and the rectangular faces: Surface area = $2 \times$ area of base + areas of 3 rectangular faces



Example 1

Determining the Surface Area of a Composite Object Made from Two Prisms

Determine the surface area of this object.





The surface area of the object is 208 cm².

When a composite object includes a right cylinder, we can use a formula to determine its surface area. A cylinder has 2 congruent bases and a curved surface. Each base is a circle, with radius *r* and area πr^2 . The curved surface is formed from a rectangle with:

- one side equal to the circumference of the circular base, and
- one side equal to the height of the cylinder

The circumference of the circular base is $2\pi r$.



Surface area = area of two circular bases + curved surface area

= 2 × area of one circular base + circumference of base × height of cylinder = 2 × πr^2 + 2 πr × height

Sometimes, one base of the cylinder is not included in the surface area calculation because the cylinder is sitting on its base. Then,

Surface area = area of one base + circumference of base × height of cylinder = $\pi r^2 + 2\pi r \times \text{height}$

Example 2

Determining the Surface Area of a Composite Object Made from Two Cylinders

Two round cakes have diameters of 14 cm and 26 cm, and are 5 cm tall. They are arranged as shown. The cakes are covered in frosting. What is the area of frosting?



Solutions

Method 1

Calculate the surface area of each cake.

Do not include the base it sits on because this will not be frosted.

The surface area is measured in square centimetres.

For the smaller cake:

The diameter is 14 cm, so the radius, *r*, is 7 cm. The height is 5 cm.

Surface area = area of one base + circumference of base \times height of cylinder

 $= \pi r^{2} + 2\pi r \times \text{height}$ = $(\pi \times 7^{2}) + (2 \times \pi \times 7 \times 5)$ Use a calculator and the order of operations. = 373.85

For the larger cake:

The diameter is 26 cm, so the radius, *r*, is 13 cm. The height is 5 cm. Surface area = area of one base + circumference of base \times height of cylinder

> $= \pi r^{2} + 2\pi r \times \text{height}$ = $(\pi \times 13^{2}) + (2 \times \pi \times 13 \times 5)$ Use a calculator. = 939.34

To calculate the area of frosting, add the two surface areas, then subtract the area of the overlap; that is, the area of the base of the smaller cake: $\pi \times 7^2$ Area of frosting $\doteq 373.85 + 939.34 - (\pi \times 7^2)$ Use a calculator.

$$\doteq 1159.25$$

The area of frosting is about 1159 cm².

Since the dimensions were given to the nearest centimetre, the surface area is given to the nearest square centimetre.

Method 2

Calculate the surface area directly.

The overlap is the area of the base of the smaller cake. So, instead of calculating the area of the top of the smaller cake, then subtracting that area as the overlap, we calculate only the curved surface area of the smaller cake.



Area of frosting = curved surface area of smaller cake + surface area of larger cake, without one base = $(2 \times \pi \times 7 \times 5) + [(\pi \times 13^2) + (2 \times \pi \times 13 \times 5)]$ Use a calculator. = 1159.25 The area of frosting is about 1159 cm².

When some of the lengths on a right triangular prism are not given, we may need to use the Pythagorean Theorem to calculate them.

Example 3 Using the Pythagorean Theorem in Surface Area Calculations

The roof, columns, and base of this porch are to be painted. The radius of each column is 20 cm. What is the area to be painted, to the nearest square metre?



A Solution

The roof is a triangular prism with its base an equilateral triangle. To determine the area of the triangular base, we need to know the height of the triangle. Let the height of the triangle be h.



The height, AD, bisects the base, BC. Use the Pythagorean Theorem in \triangle ABD. $h^2 + 1^2 = 2^2$ $h^2 + 1 = 4$ Solve for h^2 . $h^2 = 4 - 1$ = 3 $h = \sqrt{3}$ Determine the square root. $\doteq 1.732$

The height of the equilateral triangle is about 1.7 m.

Since one base of the triangular prism is against the house, it will not be painted. The rectangular faces are congruent because they have the same length and width. So, for the roof: Surface area = area of one triangular base + areas of 3 congruent rectangular faces $\doteq \left(\frac{1}{2} \times 2.0 \times 1.732\right) + [3 \times (2.0 \times 2.2)]$ = 1.732 + 13.2= 14.932

The base of the porch is a right rectangular prism with only the front, top, and 2 side faces to be painted. The units must match, so convert 15 cm to 0.15 m. Surface area = area of front face + area of top face + $2 \times$ area of side face

$$= (2.0 \times 0.15) + (2.0 \times 2.2) + [2 \times (2.2 \times 0.15)]$$

= 0.3 + 4.4 + 0.66
= 5.36

The two columns are cylinders. Only the curved surfaces need to be painted. The radius is 20 cm, which is 0.2 m.

Surface area = $2 \times (\text{circumference of base} \times \text{height of cylinder})$

= 2 × (2 π r × height) = 2 × (2 × π × 0.2 × 2.5) = 6.283

To calculate the area to be painted, add the surface areas of the roof, base, and columns, then subtract the area of overlap at the top and bottom of the columns. The area of overlap is 4 times the area of the base of one column.

The area of each circular base is:

 $\pi r^2 = \pi \times 0.2^2$ $\doteq 0.126$ Surface area = area of roof + area of base + area of cylinders $-4 \times \text{area of circular base of column}$ $\doteq 14.932 + 5.36 + 6.283 - (4 \times 0.126)$ = 26.071The area to be painted is about 26 m².



- **1.** What can you use to calculate an unknown length when the base of a right prism is a right triangle? Explain why.
- **2.** When do you think it is not helpful to draw a net to calculate the surface area of a composite object?

Practice

Check

3. Determine the surface area of each composite object. Give the answers to the nearest whole number.









c) cylinder on a cylinder



d) cube on a triangular prism



e) cube on a triangular prism



4. Determine the surface area of each composite object. Give the answers to the nearest tenth.



b) The cylinder is 3.5 m long with diameter 0.5 m.



- **5.** Determine the surface area of each composite object.
 - a) The cylinder is 2.5 m long with radius 0.5 m.



b) The base of the triangular prism is an equilateral triangle with side length 2.8 cm.



Apply

6. Here is the lamp stand from the top of page 33. The base of the lamp is a triangular prism with an equilateral triangle base. The surface of the stand is to be painted. What is the area that will be painted? Give the answer to the nearest whole number.



7. Assessment Focus

 a) A playhouse has the shape of a rectangular prism with a triangular prism roof. Determine the surface area of the playhouse.



- b) What are possible dimensions for a door and 2 windows? Explain how including these features will affect the surface area of the playhouse.
- c) Determine the surface area of the playhouse not including its doors and windows.
- **8.** Jemma has built this doghouse. The roof is a triangular prism with an isosceles triangle base. There is an overhang of 0.1 m. There is an opening for the doorway.



- a) Determine the surface area of the doghouse.
- b) The doghouse is to be covered with 2 coats of wood stain. Wood stain can be bought in 1-L or 4-L cans. One litre of stain covers 6 m². How many cans of either size are needed? Explain your thinking.
- 9. Each layer of a three-layer cake is a cylinder with height 7.5 cm. The bottom layer has diameter 25 cm. The middle layer has diameter 22.5 cm. The top layer has diameter 20 cm. The surface of the cake is frosted.
 a) Sketch the cake.
 - **b)** What area of the cake is frosted?

- **10.** In question 9, you determined the surface area of a three-layer cake.
 - a) Suppose a fourth layer, with diameter 27.5 cm, is added to the bottom of the cake. What is the surface area of cake that will be frosted now?
 - b) Suppose a fifth layer, with diameter 30 cm, is added to the bottom of the cake. What is the surface area of cake that will be frosted now?
 - c) How does the surface area change when each new layer is added?
 - Give all the answers to the nearest tenth.
- **11.** Rory will paint this birdhouse he built for his backyard. The perch is a cylinder with length 7 cm and diameter 1 cm. The diameter of the entrance is 3 cm. What is the area that needs to be painted? Give the answer to the nearest whole number.



12. Shael and Keely are camping with their parents at Waskesiu Lake in Prince Albert National Park. Their tent trailer is 5 m long and 2.5 m wide. When the trailer is set up, the canvas expands to a height of 2.5 m. At each end, there is a fold out bed that is 1.7 m wide, in a space that is shaped like a triangular prism. The diagram shows a side view of the trailer.



- a) Determine the surface area of the canvas on the trailer.
- b) Two parallel bars, 1.3 m high, are placed vertically at each end to support the canvas and provide more space in the beds. Does the surface area of the canvas change when the bars are inserted? Explain how you know.



- **13.** a) What is the surface area of a cube with edge length 24 cm?
 - b) The cube is cut along a diagonal of one face to form two triangular prisms. These prisms are glued together to form a longer triangular prism. What is the surface area of this prism? Give the answer to the nearest whole number.



c) Why do the cube and the triangular prism have different surface areas?

14. A birdbath and stand are made from 3 cylinders. The top and bottom cylinders have radius 22 cm and height 13 cm. The middle cylinder has radius 15 cm and height 40 cm. The "bath" has radius 15 cm and depth 2 cm. The birdbath and stand are to be tiled. Calculate the area to be tiled.



Take It Further

- **15.** a) What is the surface area of a cylinder that is 50 cm long and has diameter 18 cm?
 - b) The cylinder is cut in half along its length and the two pieces are glued together end to end.
 - i) Sketch the composite object.
 - ii) What is its surface area?

Give the answers to the nearest whole number.

16. Grise Fiord, Nunavut, is Canada's northernmost Inuit community and it is home to 150 residents. In Inuktitut, this hamlet is called Aujuittuq, which means "the place that never thaws." Although the ground is frozen most of the year, it softens in the summer. The freezing and thawing of the ground would ruin a house foundation. The houses are made of wood, and are built on platforms. The homes are compact and have few windows.



- a) Design and sketch the exterior of a home that could fit on a platform that is 10 m wide and 20 m long.
- **b)** Determine the surface area of this home.
- c) Every outside face needs to be insulated. Insulation costs \$4.25/m². How much will it cost to insulate this home?

Reflect

Sketch a building or structure in your community that is made up of two or more prisms or cylinders. Explain how you would determine its surface area.